TVZ TEHNIČKO VELEUČILIŠTE U ZAGREBU POLYTECHNICUM ZAGRABIENSE Elektrotehnički odjel	**Matematika 1** (preddiplomski stručni studij elektrotehnike)	**Zadaci za demonstrature** grupe **E i F** **15.12.2017.**

1. Zadana je realna funkcija $f(x) = e^{x^2+4\cdot x+3}$. Izračunajte $f^{'}(-1)$.

2. Zadana je realna funkcija $g(t) = \sin(\ln t)$. Izračunajte $g^{'}(1)$.

3. Zadana je realna funkcija $h(u) = \sqrt{6\cdot\left(1+\cos^2 u\right)}$. Izračunajte $h^{'}\left(\frac{\pi}{4}\right)$.

4. Zadana je realna funkcija $k(w) = 40\cdot \operatorname{arctg}\left(\frac{1}{\sqrt{w}}\right)$. Izračunajte $k^{'}(4)$.

5. Funkcija $y = y(x)$ definirana je implicitno jednadžbom $x^3 + y^3 = 6\cdot x\cdot y$. Izračunajte $y^{'}$ u točki $T = (3, 3)$.

6. Funkcija $x = x(t)$ definirana je implicitno jednadžbom $\ln\left(\frac{x}{t}\right)+\frac{x}{t}-t=0.$ Izračunajte $x^{'}$ u točki $T = (1, 1)$.

7. Funkcija $y = y(w)$ definirana je implicitno jednadžbom $\sin(y\cdot w)+y^2\cdot w+\ln w=0.$ Izračunajte $y^{'}$ u točki $T = (1, 0)$.

8. Funkcija $y = y(t)$ definirana je implicitno jednadžbom $2\cdot t^2+\frac{t}{y}-e^{\frac{y}{t}+1}=0$. Izračunajte $y^{'}$ u točki $T = (-1, 1)$.

9. Funkcija $y = y(x)$ zadana je parametarski s $\begin{cases} x = t^3+t^2+t+1, \\ y = t^3-t^2+t-1. \end{cases}$ Izračunajte $y^{'}$ u točki određenoj parametrom $t = 0$.

10. Funkcija $y = y(x)$ zadana je parametarski s $\begin{cases} x = t\cdot e^t, \\ y = t\cdot\cos t. \end{cases}$ Izračunajte $y^{'}$ u točki određenoj parametrom $t = 0$.

11. Funkcija $y = y(x)$ zadana je parametarski s $\begin{cases} x = 1+\sin t, \\ y = (1+\sin t)\cdot\cos t, \end{cases}$ Izračunajte $y^{'}$ u točki određenoj parametrom $t = \frac{\pi}{4}$.

12. Funkcija $y = y(x)$ zadana je parametarski s $\begin{cases} x = \cos(2\cdot t)\cdot\cos t, \\ y = \cos(2\cdot t)\cdot\sin t. \end{cases}$ Izračunajte $y^{'}$ u točki određenoj parametrom $t = \frac{3}{4}\cdot\pi$.

REZULTATI ZADATAKA

1. 2.

2. 1.

3. –1.

4. –2.

5. –1.

6. $\frac{3}{2} = 1.5$.

7. –1.

8. 1.

9. 1.

10. 1.

11. –1.

12. –1.

DETALJNIJA RJEŠENJA ZADATAKA

Funkcije u zadacima 1. – 4. treba derivirati prema pravilu deriviranja složene funkcije.

1. $f'(x) = e^{x^2+4\cdot x+3}\cdot(x^2+4\cdot x+3)' = e^{x^2+4\cdot x+3}\cdot(2\cdot x+4+0) = 2\cdot(x+2)\cdot e^{x^2+4\cdot x+3} \Rightarrow f'(-1) = 2\cdot 1\cdot e^0 = 2.$

2. $g'(t) = \cos(\ln t)\cdot(\ln t)' = \cos(\ln t)\cdot\frac{1}{t} = \frac{\cos(\ln t)}{t} \Rightarrow g'(1) = \frac{\cos(\ln 1)}{1} = \frac{\cos 0}{1} = 1.$

3. $h(u) = \left[6\cdot\left(1+\cos^2 u\right)\right]^{\frac{1}{2}} \Rightarrow h'(u) = \frac{1}{2}\cdot\left[6\cdot\left(1+\cos^2 u\right)\right]^{\frac{1}{2}-1}\cdot\left[6\cdot\left(1+\cos^2 u\right)\right]' =$

$$= \frac{1}{2}\cdot\left[6\cdot\left(1+\cos^2 u\right)\right]^{-\frac{1}{2}}\cdot 6\cdot\left(1+\cos^2 u\right)' = 3\cdot\left[6\cdot\left(1+\cos^2 u\right)\right]^{-\frac{1}{2}}\cdot\left[0+2\cdot\cos u\cdot(\cos u)'\right] =$$

$$= 3\cdot\left[6\cdot\left(1+\cos^2 u\right)\right]^{-\frac{1}{2}}\cdot 2\cdot\cos u\cdot(-\sin u) = -3\cdot\left[6\cdot\left(1+\cos^2 u\right)\right]^{-\frac{1}{2}}\cdot(2\cdot\sin u\cdot\cos u) =$$

$$= -3\cdot\left[6\cdot\left(1+\cos^2 u\right)\right]^{-\frac{1}{2}}\cdot\sin(2\cdot u) \Rightarrow h'\left(\frac{\pi}{4}\right) = -3\cdot\left\{6\cdot\left[1+\cos^2\left(\frac{\pi}{4}\right)\right]\right\}^{-\frac{1}{2}}\cdot\sin\left(2\cdot\frac{\pi}{4}\right) =$$

$$= -3\cdot\left\{6\cdot\left[1+\left(\frac{\sqrt{2}}{2}\right)^2\right]\right\}^{-\frac{1}{2}}\cdot\sin\left(\frac{\pi}{2}\right) = 3\cdot\left[6\cdot\left(1+\frac{1}{2}\right)\right]^{-\frac{1}{2}}\cdot 1 = -3\cdot 9^{-\frac{1}{2}}\cdot 1 = -1.$$

4. $k'(w) = 40\cdot\left[\operatorname{arctg}\left(\frac{1}{\sqrt{w}}\right)\right]' = 40\cdot\frac{1}{\left(\frac{1}{\sqrt{w}}\right)^2+1}\cdot\left(\frac{1}{\sqrt{w}}\right)' = 40\cdot\frac{1}{\frac{1}{w}+1}\cdot\left(w^{-\frac{1}{2}}\right)' =$

$$= 40\cdot\frac{w}{w+1}\cdot\left(-\frac{1}{2}\right)\cdot w^{-\frac{1}{2}-1} = -20\cdot\frac{w}{w+1}\cdot w^{-\frac{3}{2}} = -20\cdot\frac{w^{-\frac{1}{2}}}{w+1} = -\frac{20}{(w+1)\cdot w^{\frac{1}{2}}} = -\frac{20}{(w+1)\cdot\sqrt{w}}$$

$$\Rightarrow k'(4) = -\frac{20}{(4+1)\cdot\sqrt{4}} = -\frac{20}{5\cdot 2} = -2.$$

U zadacima 5. – 8. treba primijeniti pravilo deriviranja implicitno zadane funkcije.

5. $3\cdot x^2+3\cdot y^2\cdot y' = 6\cdot\left(1\cdot y+x\cdot 1\cdot y'\right) \Rightarrow 3\cdot x^2+3\cdot y^2\cdot y' = 6\cdot y+6\cdot x\cdot y' \Leftrightarrow$

$$\Leftrightarrow 3\cdot y^2\cdot y'-6\cdot x\cdot y' = 6\cdot y-3\cdot x^2 \Leftrightarrow y'\cdot\left(3\cdot y^2-6\cdot x\right) = 6\cdot y-3\cdot x^2 \Leftrightarrow y' = \frac{6\cdot y-3\cdot x^2}{3\cdot y^2-6\cdot x} =$$

$$= \frac{3\cdot(2\cdot y-x^2)}{3\cdot(y^2-2\cdot x)} = \frac{2\cdot y-x^2}{y^2-2\cdot x} \Rightarrow y'(3,3) = \frac{2\cdot 3-3^2}{3^2-2\cdot 3} = \frac{6-9}{9-6} = -1.$$

6. $\dfrac{1}{\frac{x}{t}}\cdot\left(\dfrac{x}{t}\right)'+\dfrac{1\cdot x'\cdot t-x\cdot 1}{t^2}-1=0\Rightarrow \dfrac{t}{x}\cdot\dfrac{x'\cdot t-x}{t^2}+\dfrac{x'\cdot t-x}{t^2}-1=0\Rightarrow$

$\Rightarrow t\cdot\left(x'\cdot t-x\right)+x\cdot\left(x'\cdot t-x\right)-x\cdot t^2=0\Rightarrow x'\cdot t^2-t\cdot x+x'\cdot t\cdot x-x^2-x\cdot t^2=0\Leftrightarrow$

$\Leftrightarrow x'\cdot t^2+x'\cdot t\cdot x=t\cdot x+x^2+x\cdot t^2\Leftrightarrow x'\cdot(t^2+x\cdot t)=x^2+x\cdot t+x\cdot t^2\Rightarrow x'=\dfrac{x^2+x\cdot t+x\cdot t^2}{t^2+x\cdot t}\Rightarrow$

$\Rightarrow x'(1,1)=\dfrac{1^2+1\cdot 1+1\cdot 1^2}{1^2+1\cdot 1}=\dfrac{3}{2}=1.5$

7. $\cos(y\cdot w)\cdot(y\cdot w)'+2\cdot y\cdot y'\cdot w+y^2\cdot 1+\dfrac{1}{w}=0\Rightarrow\cos(y\cdot w)\cdot\left(1\cdot y'\cdot w+y\cdot 1\right)+2\cdot y\cdot y'\cdot w+y^2\cdot 1+\dfrac{1}{w}=0.$

Iz ove jednakosti je nemoguće izraziti y'. Zato u nju uvrstimo $w=1,\ y=0$, pa dobijemo linearnu jednadžbu s jednom nepoznanicom $\left(y'\right)$:

$\cos(0\cdot 1)\cdot\left(1\cdot y'\cdot 1+0\cdot 1\right)+2\cdot 0\cdot y'\cdot 1+0^2\cdot 1+\dfrac{1}{1}=0\Rightarrow y'+1=0\Rightarrow y'=-1.$

8. $4\cdot t^{2-1}+\dfrac{1\cdot y-t\cdot 1\cdot y'}{y^2}-e^{\frac{y}{t}+1}\cdot\left(\dfrac{y}{t}+1\right)'=0\Rightarrow 4\cdot t+\dfrac{y-t\cdot y'}{y^2}-e^{\frac{y}{t}+1}\cdot\left(\dfrac{1\cdot y'\cdot t-y\cdot 1}{t^2}+0\right)=0\Rightarrow$

$4\cdot t+\dfrac{y-t\cdot y'}{y^2}-e^{\frac{y}{t}+1}\cdot\left(\dfrac{y'\cdot t-y}{t^2}\right)=0.$

Iz ove jednadžbe je prekomplicirano (i nepotrebno) izraziti y', pa uvrstimo $t=-1,\ y=1$:

$4\cdot(-1)+\dfrac{1-(-1)\cdot y'}{1^2}-e^{\frac{1}{-1}+1}\cdot\left[\dfrac{y'\cdot(-1)-1}{(-1)^2}\right]=0\Rightarrow -4+1+y'-1\cdot\left(-y'-1\right)=0\Rightarrow 2\cdot y'=2\Rightarrow y'=1.$

U zadacima 9. – 12. treba primijeniti pravilo deriviranja parametarski zadane funkcije.

9. $y'=\dfrac{\mathrm{d}y}{\mathrm{d}x}=\dfrac{\frac{\mathrm{d}y}{\mathrm{d}t}}{\frac{\mathrm{d}x}{\mathrm{d}t}}=\dfrac{\left(t^3-t^2+t-1\right)'}{\left(t^3+t^2+t+1\right)'}=\dfrac{3\cdot t^2-2\cdot t+1-0}{3\cdot t^2+2\cdot t+1+0}=\dfrac{3\cdot t^2-2\cdot t+1}{3\cdot t^2+2\cdot t+1}\Rightarrow\left(y'\right)_{t=0}=\dfrac{3\cdot 0^2-2\cdot 0+1}{3\cdot 0^2+2\cdot 0+1}=1.$

10. $y'=\dfrac{(t\cdot\cos t)'}{\left(t\cdot e^t\right)'}=\dfrac{1\cdot\cos t+t\cdot(-\sin t)}{1\cdot e^t+t\cdot e^t}=\dfrac{\cos t-t\cdot\sin t}{(t+1)\cdot e^t}\Rightarrow\left(y'\right)_{t=0}=\dfrac{\cos 0-0\cdot\sin 0}{(0+1)\cdot e^0}=\dfrac{1-0}{1\cdot 1}=1.$

11. $y'=\dfrac{\left[(1+\sin t)\cdot\cos t\right]'}{(1+\sin t)'}=\dfrac{(0+\cos t)\cdot\cos t+(1+\sin t)\cdot(-\sin t)}{0+\cos t}=\dfrac{(\cos^2 t-\sin^2 t)-\sin t}{\cos t}=\dfrac{\cos(2\cdot t)-\sin t}{\cos t}\Rightarrow$

$\left(y'\right)_{t=\frac{\pi}{4}}=\dfrac{\cos\left(2\cdot\frac{\pi}{4}\right)-\sin\left(\frac{\pi}{4}\right)}{\cos\left(\frac{\pi}{4}\right)}=\dfrac{\cos\left(\frac{\pi}{2}\right)-\sin\left(\frac{\pi}{4}\right)}{\cos\left(\frac{\pi}{4}\right)}=\dfrac{0-\sin\left(\frac{\pi}{4}\right)}{\cos\left(\frac{\pi}{4}\right)}=-\dfrac{\sin\left(\frac{\pi}{4}\right)}{\cos\left(\frac{\pi}{4}\right)}=-\mathrm{tg}\left(\dfrac{\pi}{4}\right)=-1.$

12. $y'=\dfrac{\left[\cos(2\cdot t)\cdot\sin t\right]'}{\left[\cos(2\cdot t)\cdot\cos t\right]'}=\dfrac{-\sin(2\cdot t)\cdot(2\cdot t)'\cdot\sin t+\cos(2\cdot t)\cdot\cos t}{-\sin(2\cdot t)\cdot(2\cdot t)'\cdot\cos t+\cos(2\cdot t)\cdot(-\sin t)}=\dfrac{\cos(2\cdot t)\cdot\cos t-2\cdot\sin(2\cdot t)\cdot\sin t}{-2\cdot\sin(2\cdot t)\cdot\cos t-\cos(2\cdot t)\cdot\sin t}\Rightarrow$

$$\left(y'\right)_{t=\frac{3}{4}\cdot\pi}=\frac{\cos\left(2\cdot\frac{3}{4}\cdot\pi\right)\cdot\cos\left(\frac{3}{4}\cdot\pi\right)-2\cdot\sin\left(2\cdot\frac{3}{4}\cdot\pi\right)\cdot\sin\left(\frac{3}{4}\cdot\pi\right)}{-2\cdot\sin\left(2\cdot\frac{3}{4}\cdot\pi\right)\cdot\cos\left(\frac{3}{4}\cdot\pi\right)-\cos\left(2\cdot\frac{3}{4}\cdot\pi\right)\cdot\sin\left(\frac{3}{4}\cdot\pi\right)}=$$

$$=\frac{\cos\left(\frac{3}{2}\cdot\pi\right)\cdot\cos\left(\frac{3}{4}\cdot\pi\right)-2\cdot\sin\left(\frac{3}{2}\cdot\pi\right)\cdot\sin\left(\frac{3}{4}\cdot\pi\right)}{-2\cdot\sin\left(\frac{3}{2}\cdot\pi\right)\cdot\cos\left(\frac{3}{4}\cdot\pi\right)-\cos\left(\frac{3}{2}\cdot\pi\right)\cdot\sin\left(\frac{3}{4}\cdot\pi\right)}=\frac{0\cdot\cos\left(\frac{3}{4}\cdot\pi\right)-2\cdot(-1)\cdot\sin\left(\frac{3}{4}\cdot\pi\right)}{-2\cdot(-1)\cdot\cos\left(\frac{3}{4}\cdot\pi\right)-0\cdot\sin\left(\frac{3}{4}\cdot\pi\right)}=$$

$$=\frac{\sin\left(\frac{3}{4}\cdot\pi\right)}{\cos\left(\frac{3}{4}\cdot\pi\right)}=\operatorname{tg}\left(\frac{3}{4}\cdot\pi\right)=-1.$$