1.1. – 1.4. ANTIDERIVACIJA I NEODREĐENI INTEGRAL. IZRAVNO (NEPOSREDNO) INTEGRIRANJE. METODA ZAMJENE. METODA DJELOMIČNE (PARCIJALNE) INTEGRACIJE. – zadaci

1.1. ANTIDERIVACIJA I NEODREĐENI INTEGRAL

1. Provjerite je li funkcija F antiderivacija funkcije f ako je:

a) $F(x) = x^2 - 2020 \cdot x + 2021,\ f(x) = 2 \cdot (x - 1010);$

b) $F(y) = e^y \cdot (\sin y + \cos y),\ f(y) = 2 \cdot e^y \cdot \cos y;$

c) $F(t) = \dfrac{\ln t}{t},\ f(t) = \dfrac{\ln t - 1}{t^2};$

d) $F(u) = \arcsin(u-1) + 2020,\ f(u) = \dfrac{1}{\sqrt{2 \cdot u - u^2}}.$

1.2. IZRAVNO (NEPOSREDNO) INTEGRIRANJE

1. Odredite sljedeće neodređene integrale koristeći osnovna svojstva, odnosno tablicu osnovnih neodređenih integrala:

a) $\int (6 \cdot x^2 - 2020 \cdot x + 2021) \cdot \mathrm{d}x;$

b) $\int \left(\dfrac{2}{t} - \dfrac{8}{t^2 - 16} + \dfrac{1}{\sqrt{t^2+1}} \right) \cdot \mathrm{d}t;$

c) $\int \left(\sin u - 2 \cdot \cos u - \dfrac{1}{2} \cdot \mathrm{tg}\, u + \mathrm{ctg}\, u \right) \cdot \mathrm{d}u;$

d) $\int \left(e^w + \dfrac{2020^w}{2} - \dfrac{2020}{w^2+1} \right) \cdot \mathrm{d}w.$

2. Odredite sljedeće neodređene integrale:

a) $\int \left(w^2 + \dfrac{1}{w} \right)^2 \cdot \mathrm{d}w;$

b) $\int \left(\sqrt[3]{y} - \dfrac{1}{\sqrt{y}} \right)^2 \cdot \mathrm{d}y;$

c) $\int \left(\sqrt{x} - \dfrac{1}{\sqrt[4]{x}} \right)^3 \cdot \mathrm{d}x;$

d) $\int \dfrac{t^2+2}{t^2+1} \cdot \mathrm{d}t.$

1.3. METODA ZAMJENE (SUPSTITUCIJE)

1. Metodom zamjene odredite sljedeće neodređene integrale:

a) $\int (2020 \cdot x - 2021)^{2022} \cdot \mathrm{d}x$;

b) $\int \frac{t \cdot \mathrm{d}t}{\sin^2(2 \cdot t^2 - 1)}$;

c) $\int \frac{\sin(2 \cdot u)}{\sqrt{4 + \sin^2 u}} \cdot \mathrm{d}u$;

d) $\int \frac{e^{\operatorname{arctg} w} + w \cdot \ln(w^2 + 1)}{w^2 + 1} \cdot \mathrm{d}w$.

2. Odredite sljedeće neodređene integrale:

a) $\int \frac{12 \cdot t^2}{\sqrt{t^6 + 1}} \cdot \mathrm{d}t$;

b) $\int \frac{x}{\sqrt{4 - x^4}} \cdot \mathrm{d}x$;

c) $\int u \cdot (2u + 1)^5 \cdot \mathrm{d}u$;

d) $\int \frac{\sin w \cdot \cos w}{\sqrt{25 - \cos^4 w}} \cdot \mathrm{d}w$.

1.4. METODA DJELOMIČNE (PARCIJALNE) INTEGRACIJE

1. Metodom djelomične integracije odredite sljedeće neodređene integrale:

a) $\int \ln x \cdot \mathrm{d}x$;

b) $\int y \cdot e^{y} \cdot \mathrm{d}y$;

c) $\int w \cdot \cos w \cdot \mathrm{d}w$;

d) $\int \operatorname{arcctg} t \cdot \mathrm{d}t$.

2. Riješite sljedeće Cauchyjeve probleme:

a) $\begin{cases} F'(x) = \sin\left(\sqrt{x}\right), \\ F(0) = 0; \end{cases}$

b) $\begin{cases} F'(y) = \ln^2 y, \\ F(1) = 2; \end{cases}$

c) $\begin{cases} F'(w) = 2 \cdot e^{w} \cdot \sin w, \\ F(0) = -1. \end{cases}$

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RJEŠENJA ZADATAKA

Napomena: U svim rješenjima $C \in \mathbb{R}$ je konstanta.

1.1. Primitivna funkcija i neodređeni integral

1.

a) $F^{'}(x) = 2 \cdot x - 2020 = 2 \cdot (x - 1010) = f(x) \Rightarrow F$ je primitivna funkcija od f;

b) $F^{'}(y) = e^{y} \cdot (\cos y + \sin y) + e^{y} \cdot (-\sin y + \cos y) = 2 \cdot e^{y} \cdot \cos y = f(y) \Rightarrow F$ je primitivna funkcija od f;

c) $F^{'}(t) = \dfrac{\frac{1}{t} \cdot t - \ln t \cdot 1}{t^2} = \dfrac{1 - \ln t}{t^2} \neq \dfrac{\ln t - 1}{t^2} = f(t) \Rightarrow F$ nije primitivna funkcija od f;

d) $F^{'}(u) = \dfrac{1}{\sqrt{1-(u-1)^2}} \cdot 1 = \dfrac{1}{\sqrt{1-u^2+2\cdot u-1}} = \dfrac{1}{\sqrt{2\cdot u-u^2}} = f(u) \Rightarrow F$ je primitivna funkcija od f.

1.2. Izravno (neposredno) integriranje

1.

a) $\int (6 \cdot x^2 - 2020 \cdot x + 2021) \cdot \mathrm{d}x = \int 6 \cdot x^2 \cdot \mathrm{d}x - \int 2020 \cdot x \cdot \mathrm{d}x + \int 2021 \cdot \mathrm{d}x = 6 \cdot \int x^2 \cdot \mathrm{d}x - 2020 \cdot \int x \cdot \mathrm{d}x + 2021 \cdot \int \mathrm{d}x =$

$= 6 \cdot \dfrac{1}{3} \cdot x^3 - 2020 \cdot \dfrac{1}{2} \cdot x^2 + 2021 \cdot x = 2 \cdot x^3 - 1010 \cdot x^2 + 2021 \cdot x + C;$

b) $\int \left(\dfrac{2}{t} - \dfrac{8}{t^2-16} + \dfrac{1}{\sqrt{t^2+1}} \right) \cdot \mathrm{d}t = \int \dfrac{2}{t} \cdot \mathrm{d}t - \int \dfrac{8}{t^2-16} \cdot \mathrm{d}t + \int \dfrac{1}{\sqrt{t^2+1}} \cdot \mathrm{d}t = 2 \cdot \int \dfrac{1}{t} \cdot \mathrm{d}t - 8 \cdot \int \dfrac{1}{t^2-4^2} \cdot \mathrm{d}t + \int \dfrac{1}{\sqrt{t^2+1}} \cdot \mathrm{d}t =$

$= 2 \cdot \ln|t| - 8 \cdot \dfrac{1}{2 \cdot 4} \cdot \ln\left|\dfrac{t+4}{t-4}\right| + \ln\left(t + \sqrt{t^2+1}\right) = 2 \cdot \ln|t| - \ln\left|\dfrac{t+4}{t-4}\right| + \ln\left(t + \sqrt{t^2+1}\right) + C;$

c) $\int \left(\sin u - 2 \cdot \cos u - \dfrac{1}{2} \cdot \mathrm{tg}\, u + \mathrm{ctg}\, u \right) \cdot \mathrm{d}u = \int \sin u \cdot \mathrm{d}u - \int 2 \cdot \cos u \cdot \mathrm{d}u - \int \dfrac{1}{2} \cdot \mathrm{tg}\, u \cdot \mathrm{d}u + \int \mathrm{ctg}\, u \cdot \mathrm{d}u = -\cos u -$

$-2 \cdot \int \cos u \cdot \mathrm{d}u - \dfrac{1}{2} \cdot \int \mathrm{tg}\, u \cdot \mathrm{d}u + \ln|\sin u| = -\cos u - 2 \cdot \sin u + \dfrac{1}{2} \cdot \ln|\cos u| + \ln|\sin u| + C;$

d) $\int \left(e^{w} + \dfrac{2020^{w}}{2} - \dfrac{2020}{w^2+1} \right) \cdot \mathrm{d}w = \int e^{w} \cdot \mathrm{d}w + \int \dfrac{2020^{w}}{2} \cdot \mathrm{d}w - \int \dfrac{2020}{w^2+1} \cdot \mathrm{d}w = e^{w} + \dfrac{1}{2} \cdot \int 2020^{w} \cdot \mathrm{d}w - 2020 \cdot \int \dfrac{1}{w^2+1} \cdot \mathrm{d}w =$

$= e^{w} + \dfrac{1}{2 \cdot \ln 2020} \cdot 2020^{w} - 2020 \cdot \mathrm{arctg}\, w + C.$

2.

a) $\int \left(w^2 + \dfrac{1}{w} \right)^2 \cdot \mathrm{d}w = \int \left(w^4 + 2 \cdot w + \dfrac{1}{w^2} \right) \cdot \mathrm{d}w = \int w^4 \cdot \mathrm{d}w + 2 \cdot \int w \cdot \mathrm{d}w + \int \dfrac{1}{w^2} \cdot \mathrm{d}w = \dfrac{1}{4+1} \cdot w^{4+1} + 2 \cdot \dfrac{1}{1+1} \cdot w^{1+1} +$

$+ \int w^{-2} \cdot \mathrm{d}w = \dfrac{1}{5} \cdot w^5 + w^2 + \dfrac{1}{-2+1} \cdot w^{-2+1} = \dfrac{1}{5} \cdot w^5 + w^2 - w^{-1} = \dfrac{1}{5} \cdot w^5 + w^2 - \dfrac{1}{w} + C;$

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b) $\int\left(\sqrt[3]{y}-\frac{1}{\sqrt{y}}\right)^2\cdot\mathrm{d}y=\int\left(y^{\frac{1}{3}}-y^{-\frac{1}{2}}\right)^2\cdot\mathrm{d}y=\int\left(y^{\frac{1}{3}\cdot 2}-2\cdot y^{\frac{1}{3}}\cdot y^{-\frac{1}{2}}+y^{-\frac{1}{2}\cdot 2}\right)\cdot\mathrm{d}y=\int\left(y^{\frac{2}{3}}-2\cdot y^{-\frac{1}{6}}+y^{-1}\right)\cdot\mathrm{d}y=$

$=\int y^{\frac{2}{3}}\cdot\mathrm{d}y-2\cdot\int y^{-\frac{1}{6}}\cdot\mathrm{d}y+\int\frac{1}{y}\cdot\mathrm{d}y=\frac{1}{\frac{2}{3}+1}\cdot y^{\frac{2}{3}+1}-2\cdot\frac{1}{-\frac{1}{6}+1}\cdot y^{-\frac{1}{6}+1}+\ln|y|=\frac{3}{5}\cdot y^{\frac{5}{3}}-\frac{12}{5}\cdot y^{\frac{5}{6}}+\ln|y|+C.$

c) $\int\left(\sqrt{x}-\frac{1}{\sqrt[4]{x}}\right)^3\cdot\mathrm{d}x=\int\left[\left(\sqrt{x}\right)^3-3\cdot\left(\sqrt{x}\right)^2\cdot\frac{1}{\sqrt[4]{x}}+3\cdot\sqrt{x}\cdot\left(\frac{1}{\sqrt[4]{x}}\right)^2-\left(\frac{1}{\sqrt[4]{x}}\right)^3\right]\cdot\mathrm{d}x=\int\left(x^{\frac{3}{2}}-3\cdot x^{1-\frac{1}{4}}+3-x^{-\frac{3}{4}}\right)\cdot\mathrm{d}x=$

$=\int x^{\frac{3}{2}}\cdot\mathrm{d}x-3\cdot\int x^{\frac{3}{4}}\cdot\mathrm{d}x+3\cdot\int\mathrm{d}x-\int x^{-\frac{3}{4}}\cdot\mathrm{d}x=\frac{1}{\frac{3}{2}+1}\cdot x^{\frac{3}{2}+1}-3\cdot\frac{1}{\frac{3}{4}+1}\cdot x^{\frac{3}{4}+1}+3\cdot x-\frac{1}{-\frac{3}{4}+1}\cdot x^{-\frac{3}{4}+1}=$

$=\frac{2}{5}\cdot x^{\frac{5}{2}}-\frac{12}{7}\cdot x^{\frac{7}{4}}+3\cdot x-4\cdot x^{\frac{1}{4}}+C;$

d) $\int\frac{t^2+2}{t^2+1}\cdot\mathrm{d}t=\int\left(1+\frac{1}{t^2+1}\right)\cdot\mathrm{d}t=\int\mathrm{d}t+\int\frac{1}{t^2+1}\cdot\mathrm{d}t=t+\operatorname{arctg}t+C.$

1.3. Metoda zamjene (supstitucije)

1.

a) $\int(2020\cdot x-2021)^{2022}\cdot\mathrm{d}x=\left\{\begin{array}{l}t=2020\cdot x-2021\\ \mathrm{d}t=2020\cdot\mathrm{d}x\\ \mathrm{d}x=\frac{1}{2020}\cdot\mathrm{d}t\end{array}\right\}=\int t^{2022}\cdot\frac{1}{2020}\cdot\mathrm{d}t=\frac{1}{2020}\cdot\int t^{2022}\cdot\mathrm{d}t=\frac{1}{2020}\cdot\frac{1}{2022+1}\cdot t^{2022+1}=$

$=\frac{1}{4086460}\cdot t^{2023}=\frac{1}{4086460}\cdot(2020\cdot x-2021)^{2023}+C;$

b) $\int\frac{t\cdot\mathrm{d}t}{\sin^2(2\cdot t^2+1)}=\left\{\begin{array}{l}u=2\cdot t^2+1\\ \mathrm{d}u=4\cdot t\cdot\mathrm{d}t\\ t\cdot\mathrm{d}t=\frac{1}{4}\cdot\mathrm{d}u\end{array}\right\}=\int\frac{\frac{1}{4}\cdot\mathrm{d}u}{\sin^2 u}=\frac{1}{4}\cdot\int\frac{1}{\sin^2 u}\cdot\mathrm{d}u=\frac{1}{4}\cdot(-\operatorname{ctg}u)=-\frac{1}{4}\cdot\operatorname{ctg}u=-\frac{1}{4}\cdot\operatorname{ctg}(2\cdot t^2+1)+C;$

c) $\int\frac{\sin(2\cdot u)}{\sqrt{4+\sin^2 u}}\cdot\mathrm{d}u=\left\{\begin{array}{l}t=4+\sin^2 u\\ \mathrm{d}t=2\cdot\sin u\cdot\cos u\cdot\mathrm{d}u=\sin(2\cdot u)\cdot\mathrm{d}u\end{array}\right\}=\int\frac{1}{\sqrt{t}}\cdot\mathrm{d}t=\int t^{-\frac{1}{2}}\cdot\mathrm{d}t=\frac{1}{-\frac{1}{2}+1}\cdot t^{-\frac{1}{2}+1}=2\cdot t^{\frac{1}{2}}=2\cdot\sqrt{t}=2\cdot\sqrt{4+\sin^2 u}+C;$

d) $\underbrace{\int\frac{e^{\operatorname{arctg}w}+w\cdot\ln(w^2+1)}{w^2+1}\cdot\mathrm{d}w}_{=I}=\underbrace{\int\frac{e^{\operatorname{arctg}w}}{w^2+1}\cdot\mathrm{d}w}_{=I_1}+\underbrace{\int\frac{w\cdot\ln(w^2+1)}{w^2+1}\cdot\mathrm{d}w}_{=I_2}.$

$I_1=\left\{\begin{array}{l}t=\operatorname{arctg}w\\ \mathrm{d}t=\frac{1}{w^2+1}\cdot\mathrm{d}w\end{array}\right\}=\int e^t\cdot\mathrm{d}t=e^t=e^{\operatorname{arctg}w};$

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$$I_2=\left\{\begin{array}{l} t=\ln(w^2+1)\\ \mathrm{d}t=\dfrac{2\cdot w}{w^2+1}\cdot \mathrm{d}w\\ \dfrac{w}{w^2+1}\cdot \mathrm{d}w=\dfrac{1}{2}\cdot \mathrm{d}t\end{array}\right\}=\int t\cdot\frac{1}{2}\cdot \mathrm{d}t=\frac{1}{2}\cdot\int t\cdot \mathrm{d}t=\frac{1}{2}\cdot\frac{1}{2}\cdot t^{1+1}=\frac{1}{4}\cdot t^2=\frac{1}{4}\cdot\ln^2(w^2+1);$$

$$I=I_1+I_2=e^{\operatorname{arctg} w}+\frac{1}{4}\cdot\ln^2(w^2+1)+C.$$

2.

a) $$\int\frac{12\cdot t^2}{\sqrt{t^6+1}}\cdot \mathrm{d}t=\left\{\begin{array}{l} x:=t^3,\\ \mathrm{d}x=3\cdot t^2\cdot \mathrm{d}t,\\ 12\cdot t^2\cdot \mathrm{d}t=4\cdot \mathrm{d}x\end{array}\right\}=\int\frac{4}{\sqrt{x^2+1}}\cdot \mathrm{d}x=4\cdot\int\frac{1}{\sqrt{x^2+1}}\cdot \mathrm{d}x=4\cdot\ln\left|x+\sqrt{x^2+1}\right|=4\cdot\ln\left|t^3+\sqrt{t^6+1}\right|+C;$$

b) $$\int\frac{x}{\sqrt{4-x^4}}\cdot \mathrm{d}x=\int\frac{x}{\sqrt{4-(x^2)^2}}\cdot \mathrm{d}x\left\{\begin{array}{l} t:=x^2\\ \mathrm{d}t=2\cdot x\cdot \mathrm{d}x\\ x\cdot \mathrm{d}x=\dfrac{1}{2}\cdot \mathrm{d}t\end{array}\right\}=\int\frac{\frac{1}{2}\cdot \mathrm{d}t}{\sqrt{4-t^2}}=\frac{1}{2}\cdot\int\frac{1}{\sqrt{2^2-t^2}}\cdot \mathrm{d}t=\frac{1}{2}\cdot\arcsin\left(\frac{t}{2}\right)=\frac{1}{2}\cdot\arcsin\left(\frac{x^2}{2}\right)+C;$$

c) $$\int u\cdot(2\cdot u+1)^5\cdot \mathrm{d}u=\left\{\begin{array}{l} t:=2\cdot u+1\\ u=\dfrac{t-1}{2}\\ \mathrm{d}t=2\cdot \mathrm{d}u\\ du=\dfrac{1}{2}\cdot \mathrm{d}t\end{array}\right\}=\int\frac{t-1}{2}\cdot t^5\cdot\frac{1}{2}\cdot \mathrm{d}t=\frac{1}{4}\cdot\int(t^6-t^5)\cdot \mathrm{d}t=\frac{1}{4}\cdot\left(\int t^6\cdot \mathrm{d}t-\int t^5\cdot \mathrm{d}t\right)=$$

$$=\frac{1}{4}\cdot\left(\frac{1}{6+1}\cdot t^{6+1}-\frac{1}{5+1}\cdot t^{5+1}\right)=\frac{1}{28}\cdot t^7-\frac{1}{24}\cdot t^6=\frac{1}{28}\cdot(2\cdot u+1)^7-\frac{1}{24}\cdot(2\cdot u+1)^6+C;$$

d) $$\int\frac{\sin w\cdot\cos w}{\sqrt{25-\cos^4 w}}\cdot \mathrm{d}w=\int\frac{\sin w\cdot\cos w}{\sqrt{25-(\cos^2 w)^2}}\cdot dw=\left\{\begin{array}{l} t:=\cos^2 w\\ \mathrm{d}t=2\cdot\cos w\cdot(-\sin w)\cdot dw\\ \sin w\cdot\cos w\cdot \mathrm{d}w=-\dfrac{1}{2}\cdot \mathrm{d}t\end{array}\right\}=\int\frac{-\frac{1}{2}\cdot \mathrm{d}t}{\sqrt{25-t^2}}=-\frac{1}{2}\cdot\int\frac{1}{\sqrt{5^2-t^2}}\cdot \mathrm{d}t=$$

$$=-\frac{1}{2}\cdot\arcsin\frac{t}{5}=-\frac{1}{2}\cdot\arcsin\left(\frac{1}{5}\cdot\cos^2 w\right)+C.$$

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1.4. Metoda djelomične (parcijalne) integracije

1.

a) $$\int \ln x \cdot \mathrm{d}x = \left\{ \begin{matrix} u = \ln x & v = \int \mathrm{d}x = x \\ \mathrm{d}u = \frac{1}{x} \cdot \mathrm{d}x & \mathrm{d}v = \mathrm{d}x \end{matrix} \right\} = x \cdot \ln x - \int x \cdot \frac{1}{x} \cdot \mathrm{d}x = x \cdot \ln x - \int \mathrm{d}x = x \cdot \ln x - x + C;$$

b) $$\int y \cdot e^y \cdot \mathrm{d}y = \left\{ \begin{matrix} u = y & v = \int e^y \cdot \mathrm{d}y = e^y \\ \mathrm{d}u = \mathrm{d}y & \mathrm{d}v = e^y \cdot \mathrm{d}y \end{matrix} \right\} = y \cdot e^y - \int e^y \cdot \mathrm{d}y = y \cdot e^y - e^y + C;$$

c) $$\int w \cdot \cos w \cdot dx = \left\{ \begin{matrix} u = w & v = \int \cos w \cdot \mathrm{d}w = \sin w \\ \mathrm{d}u = \mathrm{d}w & dv = \cos w \cdot \mathrm{d}w \end{matrix} \right\} = w \cdot \sin w - \int \sin w \cdot \mathrm{d}w = w \cdot \sin w + \cos w + C;$$

d) $$\underbrace{\int \operatorname{arcctg} t \cdot \mathrm{d}t}_{=:I} = \left\{ \begin{matrix} u = \operatorname{arcctg} t & v = \int \mathrm{d}t = t \\ \mathrm{d}u = -\frac{1}{t^2+1} \cdot \mathrm{d}t & dv = \mathrm{d}t \end{matrix} \right\} = t \cdot \operatorname{arcctg} t - \int t \cdot \left(-\frac{1}{t^2+1} \right) \cdot \mathrm{d}t = t \cdot \operatorname{arcctg} t + \underbrace{\int \frac{t}{t^2+1} \cdot \mathrm{d}t}_{=:I_1}.$$

$$I_1 = \left\{ \begin{matrix} w = t^2+1 \\ \mathrm{d}w = 2 \cdot t \cdot \mathrm{d}t \\ t \cdot \mathrm{d}t = \frac{1}{2} \cdot \mathrm{d}w \end{matrix} \right\} = \int \frac{\frac{1}{2}}{w} \cdot \mathrm{d}w = \frac{1}{2} \cdot \int \frac{1}{t} \cdot \mathrm{d}w = \frac{1}{2} \cdot \ln|w| = \frac{1}{2} \cdot \ln(t^2+1) \Rightarrow I = t \cdot \operatorname{arcctg} t + \frac{1}{2} \cdot \ln(t^2+1) + C.$$

2.

a) $$I = \int \sin\left(\sqrt{x}\right) \cdot \mathrm{d}x = \left\{ \begin{matrix} t = \sqrt{x} \\ x = t^2 \\ \mathrm{d}x = 2 \cdot t \cdot \mathrm{d}t \end{matrix} \right\} = \int \sin t \cdot 2 \cdot t \cdot \mathrm{d}t = 2 \cdot \int t \cdot \sin t \cdot \mathrm{d}t = \left\{ \begin{matrix} u = t & v = \int \sin t \cdot \mathrm{d}t = -\cos t \\ \mathrm{d}u = \mathrm{d}t & \mathrm{d}v = \sin t \cdot \mathrm{d}t \end{matrix} \right\}$$

$$= 2 \cdot \left(-t \cdot \cos t + \int \cos t \cdot \mathrm{d}t \right) = 2 \cdot (\sin t - t \cdot \cos t) = 2 \cdot \left(\sin\left(\sqrt{x}\right) - \sqrt{x} \cdot \cos\left(\sqrt{x}\right) \right) + C.$$

$$F(0) = 0 \Rightarrow 0 = 2 \cdot \left(\sin\left(\sqrt{0}\right) - \sqrt{0} \cdot \cos\left(\sqrt{0}\right) \right) + C \Rightarrow C = 0 \Rightarrow F(x) = 2 \cdot \left(\sin\left(\sqrt{x}\right) - \sqrt{x} \cdot \cos\left(\sqrt{x}\right) \right).$$

b) $$I = \int \ln^2 y \cdot \mathrm{d}y = \left\{ \begin{matrix} u = \ln^2 y & v = \int \mathrm{d}y = y \\ \mathrm{d}u = \frac{2 \cdot \ln y}{y} \cdot \mathrm{d}y & \mathrm{d}v = \mathrm{d}y \end{matrix} \right\} = y \cdot \ln^2 y - \int y \cdot \frac{2 \cdot \ln y}{y} \cdot \mathrm{d}y = y \cdot \ln^2 y - 2 \cdot \int \ln y \cdot \mathrm{d}y =$$

$$= y \cdot \left(\ln^2 y - 2 \cdot y \cdot \ln y + 2 \cdot y \right) + C.$$

$$F(1) = 2 \Rightarrow 1 \cdot \left(\ln^2 1 - 2 \cdot 1 \cdot \ln 1 + 2 \cdot 1 \right) + C = 2 \Rightarrow C + 2 = 2 \Rightarrow C = 0 \Rightarrow F(y) = y \cdot \left(\ln^2 y - 2 \cdot y \cdot \ln y + 2 \cdot y \right).$$

c) $I=\int 2\cdot e^{w}\cdot\sin w\cdot \mathrm{d}w=\left|\begin{matrix} u=2\cdot e^{w} & v=\int\sin w\cdot\mathrm{d}w=-\cos w\\ \mathrm{d}u=2\cdot e^{w}\cdot\mathrm{d}w & \mathrm{d}v=\sin w\cdot\mathrm{d}w\end{matrix}\right|=-2\cdot e^{w}\cdot\cos w+\int 2\cdot e^{w}\cdot\cos w\cdot\mathrm{d}w=$

$=\left|\begin{matrix} u=2\cdot e^{w} & v=\int\cos w\cdot\mathrm{d}w=\sin w\\ \mathrm{d}u=2\cdot e^{w}\cdot\mathrm{d}w & \mathrm{d}v=\cos w\cdot\mathrm{d}w\end{matrix}\right|=-2\cdot e^{w}\cdot\cos w+2\cdot e^{w}\cdot\sin w-\int 2\cdot e^{w}\cdot\sin w\cdot\mathrm{d}w=$

$=2\cdot e^{w}\cdot(\sin w-\cos w)-I\Rightarrow 2\cdot I=2\cdot e^{w}\cdot(\sin w-\cos w)\Rightarrow I=e^{w}\cdot(\sin w-\cos w)+C$

$F(0)=-1\Rightarrow -1=e^{0}\cdot(\sin 0-\cos 0)+C\Rightarrow C-1=-1\Rightarrow C=0\Rightarrow F(w)=e^{w}\cdot(\sin w-\cos w).$