

**1.1. – 1.4. PRIMITIVNA FUNKCIJA I NEODREĐENI INTEGRAL.
IZRAVNO (NEPOSREDNO) INTEGRIRANJE. METODA ZAMJENE.
METODA DJELOMIČNE (PARCIJALNE) INTEGRACIJE. – zadaci**

1.1. PRIMITIVNA FUNKCIJA I NEODREĐENI INTEGRAL

1. Provjerite je li funkcija F primitivna funkcija funkcije f ako je:

a) $F(x) = x^2 - 2016 \cdot x + 2017$, $f(x) = 2 \cdot (x - 1008)$;

b) $F(y) = e^y \cdot (\sin y + \cos y)$, $f(y) = 2 \cdot e^y \cdot \cos y$;

c) $F(t) = \frac{\ln t}{t}$, $f(t) = \frac{\ln t - 1}{t^2}$;

d) $F(u) = \arcsin(u - 1) + 2017$, $f(u) = \frac{1}{\sqrt{2 \cdot u - u^2}}$.

1.2. IZRAVNO (NEPOSREDNO) INTEGRIRANJE

1. Odredite sljedeće neodređene integrale koristeći osnovna svojstva, odnosno tablicu osnovnih neodređenih integrala:

a) $\int (6 \cdot x^2 - 2016 \cdot x + 2017) \cdot dx$;

b) $\int \left(\frac{2}{t} - \frac{8}{t^2 - 16} + \frac{1}{\sqrt{t^2 + 1}} \right) \cdot dt$;

c) $\int \left(\sin u - 2 \cdot \cos u - \frac{1}{2} \cdot \operatorname{tg} u + \operatorname{ctg} u \right) \cdot du$;

d) $\int \left(e^w + \frac{2017^w}{2} - \frac{2017}{w + 1} \right) \cdot dw$.

2. Odredite sljedeće neodređene integrale:

a) $\int \left(w^2 + \frac{1}{w} \right)^2 \cdot dw$;

b) $\int \left(\sqrt{x} - \frac{1}{\sqrt[4]{x}} \right)^3 \cdot dx$;

c) $\int \frac{t^2}{t^2 + 1} \cdot dt$;

d) $\int e^{u+2017} \cdot du$.

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1.3. METODA ZAMJENE (SUPSTITUCIJE)

1. Metodom zamjene odredite sljedeće neodređene integrale:

a) $\int (2016 \cdot x - 2017)^{2015} \cdot dx;$

b) $\int \frac{t \cdot dt}{\sin^2(2 \cdot t^2 - 1)};$

c) $\int \frac{\sin(2 \cdot u)}{\sqrt{4 + \sin^2 u}} \cdot du;$

d) $\int \frac{e^{\arctg w} + w \cdot \ln(w^2 + 1)}{w^2 + 1} \cdot dw.$

2. Odredite sljedeće neodređene integrale:

a) $\int \frac{x}{\sqrt{4 - x^4}} \cdot dx;$

b) $\int \sqrt{3 - 2 \cdot t - t^2} \cdot dt;$

c) $\int u \cdot (2u + 1)^5 \cdot du;$

d) $\int \frac{\sin w \cdot \cos w}{\sqrt{25 - \cos^4 w}} \cdot dw.$

1.4. METODA DJELOMIČNE (PARCIJALNE) INTEGRACIJE

1. Metodom djelomične integracije odredite sljedeće neodređene integrale:

a) $\int \ln x \cdot dx;$

b) $\int y \cdot e^y \cdot dy;$

c) $\int w \cdot \cos w \cdot dw;$

d) $\int \arctg t \cdot dt.$

2. Riješite sljedeće Cauchyjeve probleme:

a) $\begin{cases} F'(x) = e^{\sqrt{x}}, \\ F(0) = 0; \end{cases}$

b) $\begin{cases} F'(y) = \arctg \sqrt[3]{y}, \\ F(0) = 0; \end{cases}$

c) $\begin{cases} F'(w) = \sin(2 \cdot w) \cdot \ln(\sin w), \\ F\left(\frac{\pi}{2}\right) = \frac{1}{2}. \end{cases}$

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RJEŠENJA ZADATAKA

Napomena: U svim rješenjima $C \in \mathbb{R}$ je konstanta.

1.1. Primitivna funkcija i neodređeni integral

1.

a) $F'(x) = 2 \cdot x - 2016 = 2 \cdot (x - 1008) = f(x) \Rightarrow F$ je primitivna funkcija od f ;

b) $F'(y) = e^y \cdot (\cos y + \sin y) + e^y \cdot (-\sin y + \cos y) = 2 \cdot e^y \cdot \cos y = f(y) \Rightarrow F$ je primitivna funkcija od f ;

c) $F'(t) = \frac{\frac{1}{t} \cdot t - \ln t \cdot 1}{t^2} = \frac{1 - \ln t}{t^2} \neq \frac{\ln t - 1}{t^2} = f(t) \Rightarrow F$ nije primitivna funkcija od f ;

d) $F'(u) = \frac{1}{\sqrt{1 - (u-1)^2}} \cdot 1 = \frac{1}{\sqrt{1 - u^2 + 2 \cdot u - 1}} = \frac{1}{\sqrt{2 \cdot u - u^2}} = f(u) \Rightarrow F$ je primitivna funkcija od f .

1.2. Izravno (neposredno) integriranje

1.

a) $\int (6 \cdot x^2 - 2016 \cdot x + 2017) \cdot dx = \int 6 \cdot x^2 \cdot dx - \int 2016 \cdot x \cdot dx + \int 2017 \cdot dx = 6 \cdot \int x^2 \cdot dx - 2016 \cdot \int x \cdot dx + 2017 \cdot \int dx =$
 $= 6 \cdot \frac{1}{3} \cdot x^3 - 2016 \cdot \frac{1}{2} \cdot x^2 + 2017 \cdot x = 2 \cdot x^3 - 1008 \cdot x^2 + 2017 \cdot x + C;$

b) $\int \left(\frac{2}{t} - \frac{8}{t^2 - 16} + \frac{1}{\sqrt{t^2 + 1}} \right) \cdot dt = \int \frac{2}{t} \cdot dt - \int \frac{8}{t^2 - 16} \cdot dt + \int \frac{1}{\sqrt{t^2 + 1}} \cdot dt = 2 \cdot \int \frac{1}{t} \cdot dt - 8 \cdot \int \frac{1}{t^2 - 4^2} \cdot dt + \int \frac{1}{\sqrt{t^2 + 1}} \cdot dt =$
 $= 2 \cdot \ln|t| - 8 \cdot \frac{1}{2 \cdot 4} \cdot \ln \left| \frac{t+4}{t-4} \right| + \ln(t + \sqrt{t^2 + 1}) = 2 \cdot \ln|t| - \ln \left| \frac{t+4}{t-4} \right| + \ln(t + \sqrt{t^2 + 1}) + C;$

c) $\int \left(\sin u - 2 \cdot \cos u - \frac{1}{2} \cdot \operatorname{tg} u + \operatorname{ctg} u \right) \cdot du = \int \sin u \cdot du - \int 2 \cdot \cos u \cdot du - \int \frac{1}{2} \cdot \operatorname{tg} u \cdot du + \int \operatorname{ctg} u \cdot du = -\cos u -$
 $- 2 \cdot \int \cos u \cdot du - \frac{1}{2} \cdot \int \operatorname{tg} u \cdot du + \ln|\sin u| = -\cos u - 2 \cdot \sin u + \frac{1}{2} \cdot \ln|\cos u| + \ln|\sin u| + C;$

d) $\int \left(e^w + \frac{2017^w}{2} - \frac{2017}{w+1} \right) \cdot dw = \int e^w \cdot dw + \int \frac{2017^w}{2} \cdot dw - \int \frac{2017}{w+1} \cdot dw = e^w + \frac{1}{2} \cdot \int 2017^w \cdot dw - 2017 \cdot \int \frac{1}{w+1} \cdot dw =$
 $= e^w + \frac{1}{2 \cdot \ln 2017} \cdot 2017^w - 2017 \cdot \ln|w+1| + C.$

2.

a) $\int \left(w^2 + \frac{1}{w} \right)^2 \cdot dw = \int \left(w^4 + 2 \cdot w + \frac{1}{w^2} \right) \cdot dw = \int w^4 \cdot dw + \int 2 \cdot w \cdot dw + \int \frac{1}{w^2} \cdot dw = \frac{1}{4+1} \cdot w^{4+1} + 2 \cdot \int w \cdot dw +$
 $+ \int w^{-2} \cdot dw = \frac{1}{5} \cdot w^5 + 2 \cdot \frac{1}{1+1} \cdot w^{1+1} + \frac{1}{-2+1} \cdot w^{-2+1} = \frac{1}{5} \cdot w^5 + w^2 - w^{-1} = \frac{1}{5} \cdot w^5 + w^2 - \frac{1}{w} + C;$

b) $\int \left(\sqrt{x} - \frac{1}{\sqrt[4]{x}} \right)^3 \cdot dx = \int \left[\left(\sqrt{x} \right)^3 - 3 \cdot \left(\sqrt{x} \right)^2 \cdot \frac{1}{\sqrt[4]{x}} + 3 \cdot \sqrt{x} \cdot \left(\frac{1}{\sqrt[4]{x}} \right)^2 - \left(\frac{1}{\sqrt[4]{x}} \right)^3 \right] \cdot dx = \int \left(x^{\frac{3}{2}} - 3 \cdot x^{\frac{1}{4}} + 3 \cdot x^{-\frac{3}{4}} - x^{-\frac{3}{4}} \right) \cdot dx =$

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$$= \int x^{\frac{3}{2}} \cdot dx - \int 3 \cdot x^{\frac{3}{4}} \cdot dx + \int 3 \cdot dx - \int x^{\frac{3}{4}} \cdot dx = \frac{1}{\frac{3}{2}+1} \cdot x^{\frac{3}{2}+1} - 3 \cdot \int x^{\frac{3}{4}} \cdot dx + 3 \cdot \int dx - \frac{1}{-\frac{3}{4}+1} \cdot x^{-\frac{3}{4}+1} = \frac{2}{5} \cdot x^{\frac{5}{2}} - 3 \cdot \frac{1}{\frac{3}{4}+1} \cdot x^{\frac{3}{4}+1} +$$

$$+ 3 \cdot x - 4 \cdot x^{\frac{1}{4}} = \frac{2}{5} \cdot x^{\frac{5}{2}} - \frac{12}{7} \cdot x^{\frac{7}{4}} + 3 \cdot x - 4 \cdot x^{\frac{1}{4}} = \frac{2}{5} \cdot x^2 \cdot \sqrt{x} - \frac{12}{7} \cdot x \cdot \sqrt[4]{x^3} + 3 \cdot x - 4 \cdot \sqrt[4]{x} + C;$$

$$c) \int \frac{t^2}{t^2+1} \cdot dt = \int \left(1 - \frac{1}{t^2+1} \right) \cdot dt = \int dt - \int \frac{1}{t^2+1} \cdot dt = t - \arctg t + C;$$

$$d) \int e^{u+2016} \cdot du = \int e^u \cdot e^{2017} \cdot du = e^{2017} \cdot \int e^u \cdot du = e^{2017} \cdot e^u = e^{u+2017} + C.$$

1.3. Metoda zamjene (supstitucije)

1.

$$a) \int (2016 \cdot x - 2017)^{2015} \cdot dx = \left\{ \begin{array}{l} t = 2016 \cdot x - 2017 \\ dt = 2016 \cdot dx \\ dx = \frac{1}{2016} \cdot dt \end{array} \right\} = \int t^{2015} \cdot \frac{1}{2016} \cdot dt = \frac{1}{2016} \cdot \int t^{2015} \cdot dt = \frac{1}{2016} \cdot \frac{1}{2015+1} \cdot t^{2015+1} =$$

$$= \frac{1}{4064256} \cdot t^{2016} = \frac{1}{4064256} \cdot (2016 \cdot x - 2017)^{2016} + C;$$

$$b) \int \frac{t \cdot dt}{\sin^2(2 \cdot t^2 + 1)} = \left\{ \begin{array}{l} u = 2 \cdot t^2 + 1 \\ du = 4 \cdot t \cdot dt \\ t \cdot dt = \frac{1}{4} \cdot du \end{array} \right\} = \int \frac{\frac{1}{4} \cdot du}{\sin^2 u} = \frac{1}{4} \cdot \int \frac{1}{\sin^2 u} \cdot du = \frac{1}{4} \cdot (-\operatorname{ctg} u) = -\frac{1}{4} \cdot \operatorname{ctg} u = -\frac{1}{4} \cdot \operatorname{ctg}(2 \cdot t^2 + 1) + C;$$

$$c) \int \frac{\sin(2 \cdot u)}{\sqrt{4 + \sin^2 u}} \cdot du = \left\{ \begin{array}{l} t = 4 + \sin^2 u \\ dt = 2 \cdot \sin u \cdot \cos u \cdot du = \sin(2 \cdot u) \cdot du \end{array} \right\} = \int \frac{1}{\sqrt{t}} \cdot dt = \int t^{-\frac{1}{2}} \cdot dt = \frac{1}{-\frac{1}{2}+1} \cdot t^{\frac{1}{2}+1} = 2 \cdot t^{\frac{1}{2}} = 2 \cdot \sqrt{t} = 2 \cdot \sqrt{4 + \sin^2 u} + C;$$

$$d) \underbrace{\int \frac{e^{\operatorname{arctg} w} + w \cdot \ln(w^2 + 1)}{w^2 + 1} \cdot dw}_{=I} = \underbrace{\int \frac{e^{\operatorname{arctg} w}}{w^2 + 1} \cdot dw}_{=I_1} + \underbrace{\int \frac{w \cdot \ln(w^2 + 1)}{w^2 + 1} \cdot dw}_{=I_2}$$

$$I_1 = \left\{ \begin{array}{l} t = \operatorname{arctg} w \\ dt = \frac{1}{w^2 + 1} \cdot dw \end{array} \right\} = \int e^t \cdot dt = e^t = e^{\operatorname{arctg} w};$$

$$I_2 = \left\{ \begin{array}{l} t = \ln(w^2 + 1) \\ dt = \frac{2 \cdot w}{w^2 + 1} \cdot dw \\ \frac{w}{w^2 + 1} \cdot dw = \frac{1}{2} \cdot dt \end{array} \right\} = \int t \cdot \frac{1}{2} \cdot dt = \frac{1}{2} \cdot \int t \cdot dt = \frac{1}{2} \cdot \frac{1}{2} \cdot t^{1+1} = \frac{1}{4} \cdot t^2 = \frac{1}{4} \cdot \ln^2(w^2 + 1);$$

$$I = I_1 + I_2 = e^{\operatorname{arctg} w} + \frac{1}{4} \cdot \ln^2(w^2 + 1) + C$$

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2.

$$\text{a) } \int \frac{x}{\sqrt{4-x^4}} \cdot dx = \int \frac{x}{\sqrt{4-(x^2)^2}} \cdot dx \left\{ \begin{array}{l} t = x^2 \\ dt = 2 \cdot x \cdot dx \\ x \cdot dx = \frac{1}{2} \cdot dt \end{array} \right\} = \int \frac{\frac{1}{2} \cdot dt}{\sqrt{4-t^2}} = \frac{1}{2} \cdot \int \frac{1}{\sqrt{2^2-t^2}} \cdot dt = \frac{1}{2} \cdot \arcsin \frac{t}{2} = \frac{1}{2} \cdot \arcsin \frac{x^2}{2} + C;$$

$$\text{b) } \int \sqrt{3-2 \cdot t-t^2} \cdot dt = \int \sqrt{4-1-2 \cdot t-t^2} \cdot dt = \int \sqrt{2^2-(1+t)^2} \cdot dt = \int \sqrt{2^2-(t+1)^2} \cdot dx = \left\{ \begin{array}{l} u = t+1 \\ du = dt \end{array} \right\} =$$

$$= \int \sqrt{2^2-u^2} \cdot du = \frac{u}{2} \cdot \sqrt{2^2-u^2} + \frac{2^2}{2 \cdot 1} \cdot \arcsin \frac{u}{2} = \frac{u}{2} \cdot \sqrt{4-u^2} + 2 \cdot \arcsin \frac{u}{2} = \frac{t+1}{2} \cdot \sqrt{3-2 \cdot t-t^2} + 2 \cdot \arcsin \frac{t+1}{2} + C;$$

$$\text{c) } \int u \cdot (2 \cdot u+1)^5 \cdot du = \left\{ \begin{array}{l} t = 2 \cdot u+1 \\ u = \frac{t-1}{2} \\ dt = 2 \cdot du \\ du = \frac{1}{2} \cdot dt \end{array} \right\} = \int \frac{t-1}{2} \cdot t^5 \cdot \frac{1}{2} \cdot dt = \frac{1}{4} \cdot \int (t^6 - t^5) \cdot dt = \frac{1}{4} \cdot \left(\int t^6 \cdot dt - \int t^5 \cdot dt \right) =$$

$$= \frac{1}{4} \cdot \left(\frac{1}{6+1} \cdot t^{6+1} - \frac{1}{5+1} \cdot t^{5+1} \right) = \frac{1}{28} \cdot t^7 - \frac{1}{24} \cdot t^6 = \frac{1}{28} \cdot (2 \cdot u+1)^7 - \frac{1}{24} \cdot (2 \cdot u+1)^6 + C;$$

$$\text{d) } \int \frac{\sin w \cdot \cos w}{\sqrt{25-\cos^4 w}} \cdot dw = \int \frac{\sin w \cdot \cos w}{\sqrt{25-(\cos^2 w)^2}} \cdot dw = \left\{ \begin{array}{l} t = \cos^2 w \\ dt = 2 \cdot \cos w \cdot (-\sin w) \cdot dw \\ \sin w \cdot \cos w \cdot dw = -\frac{1}{2} \cdot dt \end{array} \right\} = \int \frac{-\frac{1}{2} \cdot dt}{\sqrt{25-t^2}} = -\frac{1}{2} \cdot \int \frac{1}{\sqrt{5^2-t^2}} \cdot dt =$$

$$= -\frac{1}{2} \cdot \arcsin \frac{t}{5} = -\frac{1}{2} \cdot \arcsin \left(\frac{1}{5} \cdot \cos^2 w \right) + C.$$

1.4. Metoda djelomične (parcijalne) integracije

1.

$$\text{a) } \int \ln x \cdot dx = \left\{ \begin{array}{l} u = \ln x \quad v = \int dx = x \\ du = \frac{1}{x} \cdot dx \quad dv = dx \end{array} \right\} = x \cdot \ln x - \int x \cdot \frac{1}{x} \cdot dx = x \cdot \ln x - \int dx = x \cdot \ln x - x + C;$$

$$\text{b) } \int y \cdot e^y \cdot dy = \left\{ \begin{array}{l} u = y \quad v = \int e^y \cdot dy = e^y \\ du = dy \quad dv = e^y \cdot dy \end{array} \right\} = y \cdot e^y - \int e^y \cdot dy = y \cdot e^y - e^y + C;$$

$$\text{c) } \int w \cdot \cos w \cdot dx = \left\{ \begin{array}{l} u = w \quad v = \int \cos w \cdot dw = \sin w \\ du = dw \quad dv = \cos w \cdot dw \end{array} \right\} = w \cdot \sin w - \int \sin w \cdot dw = w \cdot \sin w + \cos w + C;$$

$$\text{d) } \underbrace{\int \operatorname{arctg} t \cdot dt}_{=: I_1} = \left\{ \begin{array}{l} u = \operatorname{arctg} t \quad v = \int dt = t \\ du = \frac{1}{t^2+1} \cdot dt \quad dv = dt \end{array} \right\} = t \cdot \operatorname{arctg} t - \int t \cdot \left(-\frac{1}{t^2+1} \right) \cdot dt = t \cdot \operatorname{arctg} t + \underbrace{\int \frac{t}{t^2+1} \cdot dt}_{=: I_1}$$

$$I_1 = \left\{ \begin{array}{l} w = t^2+1 \\ dw = 2 \cdot t \cdot dt \\ t \cdot dt = \frac{1}{2} \cdot dw \end{array} \right\} = \int \frac{1}{2} \cdot dw = \frac{1}{2} \cdot \int \frac{1}{t} \cdot dw = \frac{1}{2} \cdot \ln|w| = \frac{1}{2} \cdot \ln(t^2+1) \Rightarrow I = t \cdot \operatorname{arctg} t + \frac{1}{2} \cdot \ln(t^2+1) + C.$$

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2.

$$\text{a) } I = \int e^{\sqrt{x}} \cdot dx = \left\{ \begin{array}{l} t = \sqrt{x} \\ x = t^2 \\ dx = 2 \cdot t \cdot dt \end{array} \right\} = \int e^t \cdot 2 \cdot t \cdot dt = 2 \cdot \int t \cdot e^t \cdot dt = \left\{ \begin{array}{ll} u = t & v = \int e^t \cdot dt = e^t \\ du = dt & dv = e^t \cdot dt \end{array} \right\}$$

$$= 2 \cdot \left(t \cdot e^t - \int e^t \cdot dt \right) = 2 \cdot (t - 1) \cdot e^t = 2 \cdot (\sqrt{x} - 1) \cdot e^{\sqrt{x}} + C.$$

$$F(0) = 0 \Rightarrow 0 = 2 \cdot (\sqrt{0} - 1) \cdot e^{\sqrt{0}} + C \Rightarrow C - 2 = 0 \Rightarrow C = 2.$$

$$\Rightarrow F(x) = 2 \cdot (\sqrt{x} - 1) \cdot e^{\sqrt{x}} + 2.$$

$$\text{b) } I = \int \arctg \sqrt[3]{y} \cdot dx = \left\{ \begin{array}{l} t = \sqrt[3]{y} \\ y = t^3 \\ dy = 3 \cdot t^2 \cdot dt \end{array} \right\} = \int \arctg t \cdot 3 \cdot t^2 \cdot dt = \left\{ \begin{array}{ll} u = \arctg t & v = \int 3 \cdot t^2 \cdot dt = 3 \cdot \frac{1}{2+1} \cdot t^{2+1} = t^3 \\ du = \frac{1}{t^2+1} \cdot dt & dv = 3 \cdot t^2 \cdot dt \end{array} \right\} =$$

$$= t^3 \cdot \arctg t - \int t^3 \cdot \frac{1}{t^2+1} \cdot dt = t^3 \cdot \arctg t - \int \frac{t^3}{t^2+1} \cdot dt = t^3 \cdot \arctg t - \int \left(t - \frac{t}{t^2+1} \right) \cdot dt = t^3 \cdot \arctg t - \int t \cdot dt +$$

$$\int \frac{t}{t^2+1} \cdot dt \quad \text{Primjer 1. d)} = t^3 \cdot \arctg t - \frac{1}{2} \cdot t^2 + \frac{1}{2} \cdot \ln(t^2+1) = y \cdot \arctg \sqrt[3]{y} - \frac{1}{2} \cdot \sqrt[3]{y^2} + \frac{1}{2} \cdot \ln(\sqrt[3]{y^2}+1) + C.$$

$$F(0) = 0 \Rightarrow C = 0.$$

$$\Rightarrow F(y) = y \cdot \arctg \sqrt[3]{y} - \frac{1}{2} \cdot \sqrt[3]{y^2} + \frac{1}{2} \cdot \ln(\sqrt[3]{y^2}+1).$$

$$\text{c) } I = \int \sin(2 \cdot w) \cdot \ln(\sin w) \cdot dw = \int 2 \cdot \sin w \cdot \cos w \cdot \ln(\sin w) \cdot dw = \left\{ \begin{array}{l} t = \sin w \\ dt = \cos w \cdot dw \end{array} \right\} = \int 2 \cdot t \cdot \ln t \cdot dt =$$

$$= \left\{ \begin{array}{ll} u = \ln t & v = \int 2 \cdot t \cdot dt = 2 \cdot \frac{1}{1+1} \cdot t^{1+1} = t^2 \\ du = \frac{1}{t} \cdot dt & dv = 2 \cdot t \cdot dt \end{array} \right\} = t^2 \cdot \ln t - \int t^2 \cdot \frac{1}{t} \cdot dt = t^2 \cdot \ln t - \int t \cdot dt = t^2 \cdot \ln t - \frac{1}{2} \cdot t^2 =$$

$$= \sin^2 w \cdot \ln(\sin w) - \frac{1}{2} \cdot \sin^2 w + C.$$

$$F\left(\frac{\pi}{2}\right) = \frac{1}{2} \Rightarrow C = 1.$$

$$\Rightarrow F(w) = \sin^2 w \cdot \ln(\sin w) - \frac{1}{2} \cdot \sin^2 w + 1.$$