Napomena: Ne istakne li se drugačije, pretpostavljamo da je svaki Laplaceov transformat funkcija varijable s.

1. Neka je $a \in \mathbb{R}$ konstanta. Odredite Laplaceov transformat svake od sljedećih funkcija:

a) $f:[0,+\infty) \to \mathbb{R},\ f(x)=a;$

b) $g:[0,+\infty) \to \mathbb{R},\ g(t)=e^{a\cdot t};$

c) $h:[0,+\infty) \to \mathbb{R},\ h(u)=\cos(a\cdot u);$

d) $p:[0,+\infty) \to \mathbb{R},\ p(y)=\sin(a\cdot y).$

2. Bez korištenja definicije Laplaceova transformata odredite Laplaceov transformat svake od sljedećih realnih funkcija:

a) $f(x)=2018\cdot(1+x+e^{-x});$

b) $g(t)=t^3+e^{2\cdot t}\cdot(\cos t-\sin t);$

c) $h(y)=y\cdot\sin y-e^{y}\cdot\left[\cos(2\cdot y)-\sin(4\cdot y)\right];$

d) $p(w)=\mathrm{ch}(3\cdot w)-\mathrm{sh}(2\cdot w).$

3. **a)** Odredite Laplaceov transformat realne funkcije $f(x)=x\cdot\sin x$.

b) Isključivo koristeći rješenje **a)** podzadatka i odgovarajuća svojstva Laplaceove transformacije odredite Laplaceov transformat sljedećih realnih funkcija:

$$f_1(x)=x\cdot\cos x+\sin x,$$
$$f_2(x)=2\cdot\cos x-x\cdot\sin x,$$
$$f_3(x)=6\cdot x\cdot e^{5\cdot x}\cdot\sin x.$$

4. Odredite original svakoga od sljedećih Laplaceovih transformata (pretpostavite da je original funkcija varijable x):

a) $F(s)=\dfrac{s^3+s^2-2s+6}{s^4};$

b) $F(s)=\dfrac{4\cdot s}{s^4+8\cdot s^2+16};$

c) $F(s)=\dfrac{s-2}{s^2-2\cdot s+2};$

d) $F(s)=\dfrac{2\cdot(s-2)}{s^3-s};$

e) $F(s)=\dfrac{4\cdot(3-s)}{s^2-4};$

f) $F(s)=\dfrac{s^3-4\cdot s^2-8}{s^4+4\cdot s^2}.$

RJEŠENJA ZADATAKA

1.

a) $\int f(s)\cdot e^{-s\cdot x}\cdot \mathrm{d}x=\int a\cdot e^{-s\cdot x}\cdot \mathrm{d}x=-\frac{a}{s\cdot e^{s\cdot x}}+C\Rightarrow \int_0^{+\infty} f(s)\cdot e^{-s\cdot x}\cdot \mathrm{d}x=\lim_{x\to+\infty}\left(-\frac{a}{s\cdot e^{s\cdot x}}\right)+\frac{a}{s}=\frac{a}{s}\Rightarrow L(f)=\frac{a}{s};$

b) $\int g(t)\cdot e^{-s\cdot t}\cdot \mathrm{d}t=\int e^{(a-s)\cdot t}\cdot \mathrm{d}t=\frac{1}{(s-a)\cdot e^{-(s-a)\cdot t}}+C\Rightarrow L(g)=\frac{1}{s-a};$

c) $\int h(u)\cdot e^{-s\cdot x}\cdot \mathrm{d}u=\int \sin(a\cdot u)\cdot e^{-s\cdot u}\cdot \mathrm{d}u=-\frac{s\cdot\sin(a\cdot u)+a\cdot\cos(a\cdot u)}{(s^2+a^2)\cdot e^{s\cdot u}}+C\Rightarrow L(h)=\frac{a}{s^2+a^2};$

d) $\int p(y)\cdot e^{-s\cdot y}\cdot \mathrm{d}y=\int \cos(a\cdot y)\cdot e^{-s\cdot y}\cdot \mathrm{d}y=\frac{a\cdot\sin(a\cdot y)-s\cdot\cos(a\cdot y)}{(s^2+a^2)\cdot e^{s\cdot y}}+C\Rightarrow L(p)=\frac{s}{s^2+a^2}.$

2.

a) $f(x)=2018\cdot(1+x+e^{-x})\Rightarrow F(s)=2018\cdot\left(\frac{1}{s}+\frac{1}{s^2}+\frac{1}{s-1}\right)=\frac{2018\cdot(2\cdot s^2-1)}{s^3-s^2};$

b) $g(t)=t^3+e^{2\cdot t}\cdot(\cos t-\sin t)\Rightarrow F(s)=\frac{s-3}{s^2-4\cdot s+5}+\frac{6}{s^4}=\frac{s^5-3\cdot s^4+6\cdot s^2-24\cdot s+30}{s^6-4\cdot s^5+5\cdot s^4};$

c) $h(y)=y^2\cdot\sin y-e^y\cdot\left[\cos(2\cdot y)-\sin(2\cdot y)\right]\Rightarrow F(s)=\frac{s-3}{s^2-2\cdot s+5}-\frac{2}{(s^2+1)^2}+\frac{8\cdot s^2}{(s^2+1)^3};$

d) $p(w)=\mathrm{ch}\,(3\cdot w)-\mathrm{sh}(2\cdot w)\Rightarrow F(s)=\frac{s}{s^2-9}-\frac{2}{s^2-4}.$

3.

a) $F(s)=\frac{2\cdot s}{(s^2+1)^2}.$

b) $f_1=f'\Rightarrow F_1(s)=s\cdot F(s)-f(0)=\frac{2\cdot s^2}{(s^2+1)^2};$

$f_2=f''\Rightarrow F_2(s)=s^2\cdot F(s)-s\cdot f(0)-f'(0)=\frac{2\cdot s^3}{(s^2+1)^2};$

$f_3=6\cdot e^{5\cdot x}\cdot f\Rightarrow F_5(s)=6\cdot F(s-5)=\frac{12\cdot s-60}{(s^2-10\cdot s+26)^2}.$

4.

a) $F(s)=\frac{s^3+s^2-2s+6}{s^4}=\frac{1}{s}+\frac{1}{s^2}-\frac{2}{s^3}+\frac{6}{s^4}\Rightarrow f(x)=x^3-x^2+x+1;$

b) $F(s)=\frac{4\cdot s}{s^4+8\cdot s^2+16}=\frac{2\cdot 2\cdot s}{(s^2+2^2)^2}\Rightarrow f(x)=x\cdot\sin(2\cdot x);$

c) $F(s)=\frac{s-2}{s^2-2\cdot s+2}=\frac{s-1}{(s-1)^2+1}-\frac{1}{(s-1)^2+1}\Rightarrow f(x)=e^x\cdot(\cos x-\sin x);$

d) $F(s)=\dfrac{2\cdot(s-2)}{s^3-s}=\dfrac{4}{s}-\dfrac{1}{s-1}-\dfrac{3}{s+1}\Rightarrow f(x)=4-e^{x}-3\cdot e^{-x};$

e) $F(s)=\dfrac{4\cdot(3-s)}{s^2-4}=\dfrac{1}{s-2}-\dfrac{5}{s+2}\Rightarrow f(x)=e^{2\cdot x}-5\cdot e^{-2\cdot x};$

f) $F(s)=\dfrac{s^3-4\cdot s^2-8}{s^4+4\cdot s^2}=\dfrac{s-2}{s^2+4}-\dfrac{2}{s^2}=\dfrac{s}{s^2+2^2}-\dfrac{2}{s^2+2^2}-\dfrac{2}{s}\Rightarrow f(x)=\cos(2\cdot x)-\sin(2\cdot x)-2\cdot x.$