Isključivo koristeći Laplaceovu transformaciju riješite sljedeće Cauchyjeve probleme:

**1.** $\begin{cases} 3\cdot y'' + 2\cdot y' = 4\cdot x, \\ y(0) = 2, \\ y'(0) = -3. \end{cases}$

**2.** $\begin{cases} 2\cdot y'' - 5\cdot y' + 2\cdot y = 3\cdot e^{2\cdot t}, \\ y(0) = 0, \\ y'(0) = 1. \end{cases}$

**3.** $\begin{cases} y'' - y' - 2\cdot y = 2\cdot(1-3\cdot u)\cdot e^{-u}, \\ y(0) = y'(0) = 0. \end{cases}$

**4.** $\begin{cases} y'' + y' + 16\cdot y = 4\cdot\cos(4\cdot w), \\ y(0) = 0, \\ y'(0) = 4. \end{cases}$

**5.** $\begin{cases} y'' - y' + 4\cdot y = 4\cdot\sin(2\cdot z), \\ y(0) = 2, \\ y'(0) = 0. \end{cases}$

**6.** $\begin{cases} y'' + y = \cos\alpha, \\ y(0) = 1, \\ y'(0) = 0. \end{cases}$

**7.** $\begin{cases} y'' + 4\cdot y + 4\cdot\cos(2\cdot\beta) = 0, \\ y(0) = 0, \\ y'(0) = -1. \end{cases}$

**8.** $\begin{cases} y'' - y' - y = 3\cdot e^{2\cdot\varepsilon}\cdot\cos\varepsilon, \\ y(0) = 0, \\ y'(0) = 1. \end{cases}$

**9.** $\begin{cases} y'' + 5\cdot y' + 3\cdot y + 21\cdot e^{\phi}\cdot\sin(3\cdot\phi) = 0, \\ y(0) = y'(0) = 1. \end{cases}$

**10.** $\begin{cases} y'' - y = 8\cdot\mathrm{ch}(3\cdot\varphi), \\ y(0) = y'(0) = 1. \end{cases}$

**11.** $\begin{cases} 2\cdot y'' + 3\cdot y' + 2\cdot y = e^{-\gamma}, \\ y(0) = 1, \\ y'(0) = -1. \end{cases}$

**12.** $\begin{cases} y'' - y = 2\cdot e^{\eta}, \\ y(0) = 4, \\ y'(0) = 5. \end{cases}$

## RJEŠENJA ZADATAKA

**1.** $3\cdot\left[s^2\cdot F(s)-s\cdot y(0)-y'(0)\right]+2\cdot\left[s\cdot F(s)-y(0)\right]=4\cdot\frac{1}{s^2},$

$$3\cdot\left[s^2\cdot F(s)-2\cdot s+3\right]+2\cdot\left[s\cdot F(s)-2\right]=\frac{4}{s^2},$$

$$F(s)\cdot(3\cdot s^2+2\cdot s)=\frac{4}{s^2}+6\cdot s-5,$$

$$F(s)=\frac{6\cdot s^3-5\cdot s^2+4}{s^3\cdot(3\cdot s+2)}=\frac{2\cdot s^2-3\cdot s+2}{s^3}=\frac{2}{s}-\frac{3}{s^2}+\frac{2}{s^3}$$

$$\Rightarrow y=2-3\cdot x+x^2 \Leftrightarrow y=x^2-3\cdot x+2.$$

**2.** $2\cdot\left[s^2\cdot F(s)-s\cdot y(0)-y'(0)\right]-5\cdot\left[s\cdot F(s)-y(0)\right]+2\cdot F(s)=3\cdot\frac{1}{s-2},$

$$2\cdot\left[s^2\cdot F(s)-1\right]-5\cdot s\cdot F(s)+2\cdot F(s)=\frac{3}{s-2},$$

$$F(s)\cdot(2\cdot s^2-5\cdot s+2)=\frac{3}{s-2}+2,$$

$$F(s)=\frac{2\cdot s-1}{(s-2)\cdot(2\cdot s^2-5\cdot s+2)}=\frac{2\cdot s-1}{(s-2)\cdot(s-2)\cdot(2\cdot s-1)}=\frac{1}{(s-2)^2}$$

$$\Rightarrow y=t\cdot e^{2\cdot t}.$$

**3.** $s^2\cdot F(s)-s\cdot y(0)-y'(0)-\left[s\cdot F(s)-y(0)\right]-2\cdot F(s)=2\cdot\frac{1}{s+1}-6\cdot\frac{1}{(s+1)^2},$

$$s^2\cdot F(s)-s\cdot F(s)-2\cdot F(s)=\frac{2\cdot(s+1)-6}{(s+1)^2},$$

$$F(s)\cdot(s^2-s-2)=\frac{2\cdot(s-2)}{(s+1)^2},$$

$$F(s)=\frac{2\cdot(s-2)}{(s+1)^2\cdot(s^2-s-2)}=\frac{2\cdot(s-2)}{(s+1)^3\cdot(s-2)}=\frac{2}{(s+1)^3}$$

$$\Rightarrow y=u^2\cdot e^{-u}.$$

**4.** $s^2\cdot F(s)-s\cdot y(0)-y'(0)+\left[s\cdot F(s)-y(0)\right]+16\cdot F(s)=4\cdot\frac{s}{s^2+4^2}$

$$s^2\cdot F(s)-4+s\cdot F(s)+16\cdot F(s)=\frac{4\cdot s}{s^2+16}$$

$$F(s)\cdot(s^2+s+16)=\frac{4\cdot(s^2+s+16)}{s^2+16},$$

$$F(s)=\frac{4}{s^2+16}=\frac{4}{s^2+4^2}$$

$$\Rightarrow y=\sin(4\cdot w)$$

**5**. $s^2 \cdot F(s) - s \cdot y(0) - y'(0) - [s \cdot F(s) - y(0)] + 4 \cdot F(s) = 4 \cdot \frac{2}{s^2 + 2^2}$,

$$s^2 \cdot F(s) - 2 \cdot s - s \cdot F(s) + 2 + 4 \cdot F(s) = \frac{8}{s^2 + 4},$$

$$F(s) \cdot (s^2 - s + 4) = \frac{2 \cdot s \cdot (s^2 - s + 4)}{s^2 + 4},$$

$$F(s) = \frac{2 \cdot s}{s^2 + 4} = 2 \cdot \frac{s}{s^2 + 2^2}$$

$$\Rightarrow y = 2 \cdot \cos(2 \cdot z)$$

**6**. $s^2 \cdot F(s) - s \cdot y(0) - y'(0) + F(s) = \frac{s}{s^2 + 1^2}$,

$$s^2 \cdot F(s) - s + F(s) = \frac{s}{s^2 + 1},$$

$$F(s) \cdot (s^2 + 1) = \frac{s}{s^2 + 1} + s,$$

$$F(s) = \frac{s}{s^2 + 1} + \frac{s}{(s^2 + 1)^2} = \frac{s}{s^2 + 1} + \frac{1}{2} \cdot \frac{2 \cdot 1 \cdot s}{(s^2 + 1)^2}$$

$$\Rightarrow y = \cos\alpha + \frac{1}{2} \cdot \alpha \cdot \sin\alpha.$$

**7**. $s^2 \cdot F(s) - s \cdot y(0) - y'(0) + 4 \cdot F(s) = (-4) \cdot \frac{s}{s^2 + 2^2}$,

$$s^2 \cdot F(s) + 1 + 4 \cdot F(s) = \frac{(-4) \cdot s}{s^2 + 4},$$

$$F(s) \cdot (s^2 + 4) = \frac{(-4) \cdot s}{s^2 + 4} - 1,$$

$$F(s) = -\frac{4 \cdot s}{(s^2 + 4)^2} - \frac{1}{s^2 + 4} = -\frac{2 \cdot 2 \cdot s}{(s^2 + 2^2)^2} - \frac{1}{2} \cdot \frac{2}{s^2 + 2^2}$$

$$\Rightarrow y = -\beta \cdot \sin(2 \cdot \beta) - \frac{1}{2} \cdot \sin(2 \cdot \beta) = -(2 \cdot \beta + 1) \cdot \sin\beta \cdot \cos\beta.$$

**8**. $s^2 \cdot F(s) - s \cdot y(0) - y'(0) - [s \cdot F(s) - y(0)] - F(s) = 3 \cdot \frac{s - 2}{(s - 2)^2 + 1^2}$,

$$F(s) \cdot (s^2 - s - 1) = \frac{3 \cdot (s - 2)}{(s - 2)^2 + 1} + 1,$$

$$F(s) \cdot (s^2 - s - 1) = \frac{s^2 - s - 1}{(s - 2)^2 + 1},$$

$$F(s) = \frac{1}{(s - 2)^2 + 1^2}$$

$$\Rightarrow y = e^{2 \cdot \varepsilon} \cdot \sin\varepsilon.$$

**9**. $s^2 \cdot F(s) - s \cdot y(0) - y'(0) + 5 \cdot \left[s \cdot F(s) - y(0)\right] + 3 \cdot F(s) = (-21) \cdot \dfrac{3}{(s-1)^2 + 3^2}$,

$$s^2 \cdot F(s) - s - 1 + 5 \cdot s \cdot F(s) - 5 + 3 \cdot F(s) = \frac{-63}{(s-1)^2 + 3^2},$$

$$F(s) \cdot (s^2 + 5 \cdot s + 3) = \frac{-63}{(s-1)^2 + 9} + s + 6,$$

$$F(s) \cdot (s^2 + 5 \cdot s + 3) = \frac{s^3 + 4 \cdot s^2 - 2 \cdot s - 3}{(s-1)^2 + 9},$$

$$F(s) = \frac{s^3 + 4 \cdot s^2 - 2 \cdot s - 3}{\left[(s-1)^2 + 9\right] \cdot (s^2 + 5 \cdot s + 3)} = \frac{s-1}{(s-1)^2 + 3^2}$$

$$\Rightarrow y = e^{\phi} \cdot \cos(3 \cdot \phi).$$

**10**. $s^2 \cdot F(s) - s \cdot y(0) - y'(0) - F(s) = 8 \cdot \dfrac{s}{s^2 - 9}$,

$$s^2 \cdot F(s) - s - 1 - F(s) = 8 \cdot \frac{s}{s^2 - 9},$$

$$F(s) \cdot (s^2 - 1) = \frac{8 \cdot s}{s^2 - 9} + s + 1,$$

$$F(s) = \frac{s^3 + s^2 - s - 9}{(s^2 - 9) \cdot (s^2 - 1)} = \frac{s^2 - 9}{(s^2 - 9) \cdot (s^2 - 1)} + \frac{s \cdot (s^2 - 1)}{(s^2 - 9) \cdot (s^2 - 1)} = \frac{1}{s^2 - 1} + \frac{s}{s^2 - 3^2}$$

$$\Rightarrow y = \operatorname{sh} \varphi + \operatorname{ch}(3 \cdot \varphi).$$

**11**. $2 \cdot \left[s^2 \cdot F(s) - s \cdot y(0) - y'(0)\right] + 3 \cdot \left[s \cdot F(s) - y(0)\right] + 2 \cdot F(s) = \dfrac{1}{s+1}$,

$$2 \cdot s^2 \cdot F(s) - 2 \cdot s + 2 + 3 \cdot s \cdot F(s) - 3 + 2 \cdot F(s) = \frac{1}{s+1},$$

$$F(s) \cdot (2 \cdot s^2 + 3 \cdot s + 2) = \frac{1}{s+1} + 2 \cdot s + 1,$$

$$F(s) = \frac{2 \cdot s^2 + 3 \cdot s + 2}{(2 \cdot s^2 + 3 \cdot s + 2) \cdot (s+1)} = \frac{1}{s+1}$$

$$\Rightarrow y = e^{-\gamma}.$$

**12**. $s^2 \cdot F(s) - s \cdot 4 - 5 - F(s) = \dfrac{2}{s-1}$,

$$F(s) \cdot (s^2 - 1) = \frac{2}{s-1} + 4 \cdot s + 5,$$

$$F(s) = \frac{2 + 4 \cdot s^2 + 5 \cdot s - 4 \cdot s - 5}{(s-1) \cdot (s^2 - 1)} = \frac{4 \cdot s^2 + s - 3}{(s-1) \cdot (s-1) \cdot (s+1)} = \frac{(s+1) \cdot (4 \cdot s - 3)}{(s-1) \cdot (s-1) \cdot (s+1)},$$

$$F(s) = \frac{4 \cdot s - 3}{(s-1)^2} = \frac{1}{(s-1)^2} + 4 \cdot \frac{1}{s-1},$$

$$\Rightarrow y = \eta \cdot e^{\eta} + 4 \cdot e^{\eta}.$$