REALNI I KOMPLEKSNI BROJEVI. ALGEBARSKE OPERACIJE S KOMPLEKSNIM BROJEVIMA.

1. Izračunajte $z_1 + z_2$, $z_1 - z_2$, $z_2 - z_1$, $z_1 \cdot z_2$, $\frac{z_1}{z_2}$, $\frac{z_2}{z_1}$, $|z_1|$, $|z_2|$, $|z_1 + z_2|$, $|z_1 - z_2|$, $|z_1 \cdot z_2|$, $\left|\frac{z_1}{z_2}\right|$ i $\left|\frac{z_2}{z_1}\right|$ ako je zadano:

a) $z_1 = 1 + i$, $z_2 = -i$;
b) $z_1 = 1 - i$, $z_2 = i$;
c) $z_1 = i$, $z_2 = 1 + i$;
d) $z_1 = -i$, $z_2 = 1 - i$;
e) $z_1 = 2 - i$, $z_2 = 2 + i$;
f) $z_1 = 5 - 12 \cdot i$, $z_2 = -2 + 8 \cdot i$
g) $z_1 = \frac{1}{2} - \frac{1}{3} \cdot i$, $z_2 = \frac{1}{3} + \frac{1}{2} \cdot i$;
h) $z_1 = \frac{2}{3} + \frac{4}{5} \cdot i$, $z_2 = \frac{5}{4} - \frac{3}{2} \cdot i$;
i) $z_1 = \frac{\sqrt{3}}{2} - \frac{1}{2} \cdot i$, $z_2 = \frac{1}{2} + \frac{\sqrt{3}}{2} \cdot i$;
j) $z_1 = \frac{\sqrt{3}}{2} - \frac{1}{2} \cdot i$, $z_2 = \frac{1}{2} + \frac{\sqrt{3}}{2} \cdot i$;
k) $z_1 = \frac{2\sqrt{2}}{3} + \frac{1}{3} \cdot i$, $z_2 = \frac{1}{3} - \frac{2\sqrt{2}}{3} \cdot i$;
l) $z_1 = -\frac{2\sqrt{5}}{5} - \frac{4}{5} \cdot i$, $z_2 = -\frac{\sqrt{5}}{5} + \frac{2}{5} \cdot i$;
m) $z_1 = \frac{3\sqrt{2}}{2} + \frac{\sqrt{7}}{2} \cdot i$, $z_2 = -\frac{\sqrt{7}}{2} - \frac{\sqrt{2}}{2} \cdot i$;
n) $z_1 = \frac{1+i}{2 \cdot i}$, $z_2 = -\frac{i}{i+1}$;
o) $z_1 = \frac{(1-i)^2}{1+i}$, $z_2 = \frac{(1+i)^2}{i-1}$;
p) $z_1 = (3 - 4 \cdot i)^2$, $z_2 = (-4 + 3 \cdot i)^2$;
q) $z_1 = (1 - 2 \cdot i)^3 + 13$, $z_2 = (2 + 3 \cdot i) + 58$;
r) $z_1 = (2 + i)^3 + (1 - 2 \cdot i)^3 - i$, $z_2 = (2 - i)^3 - (1 + 2 \cdot i)^3 - 1$;
s) $z_1 = (-1 + i)^3 \cdot (1 - i)^2 - 1$, $z_2 = (-1 - i)^3 \cdot (1 + i)^2 - i$;
t) $z_1 = (2 + i) \cdot (3 - i) - 5$, $z_2 = (2 - i) \cdot (3 + i) + 2 \cdot i$;
u) $z_1 = (4 - 5 \cdot i) \cdot (3 - 2 \cdot i) - (5 \cdot i)^2$, $z_2 = (5 + 4 \cdot i) \cdot (2 + 3 \cdot i) + (5 \cdot i)^2$;
v) $z_1 = (6 + 5 \cdot i) \cdot (7 + 8 \cdot i) + (9 \cdot i)^2$, $z_2 = (5 - 6 \cdot i) \cdot (8 - 7 \cdot i) - (9 \cdot i)^2$;
w) $z_1 = (1 - i) \cdot (2 + i) \cdot (3 - i)$, $z_2 = (-1 + i) \cdot (-2 - i) \cdot (-3 + i)$;

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x) $z_1 = \dfrac{(1-\sqrt{3}\cdot i)^3}{8}$, $z_2 = \dfrac{(1+\sqrt{3}\cdot i)^3}{8}$;

y) $z_1 = \dfrac{(1-\sqrt{2}\cdot i)\cdot(\sqrt{2}\cdot i+1)}{6}$, $z_2 = \dfrac{(\sqrt{2}-i)\cdot(i+\sqrt{2})}{6}$;

z) $z_1 = \dfrac{(\sqrt[3]{2}+i)\cdot(\sqrt[3]{4}-1-\sqrt[3]{2}\cdot i)}{2+i}$, $z_2 = \dfrac{(\sqrt[3]{2}-i)\cdot(\sqrt[3]{4}-1+\sqrt[3]{2}\cdot i)}{2-i}$.

2. Odredite točke Gaussove ravnine pridružene kompleksnim brojevima z i $\overline{z}$, pa ih prikažite grafički u pravokutnom koordinatnom sustavu u toj ravnini ako je zadano:

a) $z = i$;
b) $z = -2\cdot i$;
c) $z = 1$;
d) $z = -1$;
e) $z = 2 + 3\cdot i$;
f) $z = -3 + 2\cdot i$;
g) $z = -2 - 3\cdot i$;
h) $z = 3 - 2\cdot i$;
i) $z = \overline{5+i}$;
j) $z = \overline{1-3\cdot i}$;
k) $z = 2\cdot\overline{-i}$;
l) $z = (-2)\cdot\overline{(-2)\cdot i}$;
m) $z = 2\cdot\overline{\left(\dfrac{1}{2}+2\cdot i\right)}$;
n) $z = (-4)\cdot\overline{\left(-\dfrac{1}{2}+\dfrac{1}{4}\cdot i\right)}$;
o) $z = (-5)\cdot\overline{\left(-\dfrac{1}{10}-\dfrac{1}{5}\cdot i\right)}$;
p) $z = \dfrac{1}{2}\cdot\overline{(-1-2\cdot i)}$;
q) $z = \left(-\dfrac{1}{2}\right)\cdot\overline{(3+2\cdot i)}$;
r) $z = \sqrt{2}\cdot\overline{\left(\dfrac{1}{2}\cdot\sqrt{8}-\dfrac{\sqrt{2}}{2}\cdot i\right)}$;
s) $z = (5 - 2\cdot i)\cdot(2 - 5\cdot i) - (5\cdot i)^2$;
t) $z = (7 + 8\cdot i)\cdot(8 + 7\cdot i) + (4\cdot\sqrt{7}\cdot i)^2$;
u) $z = \dfrac{3-i}{2+i}+\dfrac{3+i}{2-i}$;

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v) $z = 4 \cdot \frac{1 - 2 \cdot i}{2 + i} - 3 \cdot \frac{2 - i}{1 + 2 \cdot i} + 2 \cdot i$;

w) $z = 10 \cdot \left(\frac{2 + i}{3 + i} - \frac{3 - i}{2 - i} \right) + 6$;

x) $z = \left(-\frac{9}{4} \right) \cdot \left(\frac{2\sqrt{2} + i}{\sqrt{2} - i} - \frac{3\sqrt{2} + i}{2\sqrt{2} - i} \right) + \sqrt{2} \cdot i$

y) $z = 25 \cdot \overline{\left[\frac{(5 + 3 \cdot i) \cdot (4 - 7 \cdot i)}{3 + 4 \cdot i} \right]} - \overline{4 \cdot (8 - 29 \cdot i)}$

z) $z = \frac{125}{82} \cdot \left\{ \overline{\left[\left(\frac{1 - 2 \cdot i}{1 + 2 \cdot i} \right)^2 \right]} + \overline{\left[\left(\frac{1 + 2 \cdot i}{1 - 2 \cdot i} \right)^3 \right]} \right\}$.

3. Odredite brojeve $x, y \in \mathbb{R}$ tako da kompleksni brojevi z_1 i z_2 budu jednaki ako je zadano:

a) $z_1 = x + i,\ z_2 = 1 + y \cdot i$;
b) $z_1 = x^2 - i,\ z_2 = 4 + y^3 \cdot i$;
c) $z_1 = x - 2 \cdot i,\ z_2 = 2 \cdot y \cdot i$;
d) $z_1 = x^2 - 5 \cdot x + 6 + (y^2 + 3 \cdot y - 1) \cdot i,\ z_2 = (-3) \cdot i$;
e) $z_1 = x,\ z_2 = -y \cdot i$;
f) $z_1 = -\sqrt{2} \cdot x \cdot i,\ z_2 = \sqrt[3]{2014} \cdot y$;
g) $z_1 = x + y \cdot i,\ z_2 = 2 - i$;
h) $z_1 = x - y \cdot i,\ z_2 = -5 + i$;
i) $z_1 = 2 \cdot x - 5 \cdot i,\ z_2 = -4 + 5 \cdot y \cdot i$;
j) $z_1 = x + y + (x - y) \cdot i,\ z_2 = 4 - 2 \cdot i$;
k) $z_1 = x + y - (x - y) \cdot i,\ z_2 = 2 + 4 \cdot i$;
l) $z_1 = x - y + (x + y) \cdot i,\ z_2 = -1 - 3 \cdot i$;
m) $z_1 = x - y - (x + y) \cdot i,\ z_2 = -5 + 7 \cdot i$;
n) $z_1 = 6 \cdot x + y - (x - 3 \cdot y) \cdot i,\ z_2 = (5 + i) \cdot (7 + 2 \cdot i)$;
o) $z_1 = x - y - (3 \cdot y - 4 \cdot x) \cdot i,\ z_2 = (7 - i) \cdot (2 - 5 \cdot i)$;
p) $z_1 = x + 10 \cdot y - (x - 11 \cdot y) \cdot i,\ z_2 = 25 \cdot \frac{9 - i}{3 + 4 \cdot i} + 16$;
q) $z_1 = 2 \cdot x - y + (x - 2 \cdot y) \cdot i,\ z_2 = 2 \cdot \frac{4 - 3 \cdot i}{6 + 8 \cdot i} + 1$;
r) $z_1 = (x^2 - y) + (x - y^2) \cdot i,\ z_2 = 0$;
s) $z_1 = (x^2 - y^2) + (x + y) \cdot i,\ z_2 = 2 \cdot \overline{6 - i}$;
t) $z_1 = (x^2 - y^2) - (x - y) \cdot i,\ z_2 = 3 \cdot \overline{-1 - i}$;
u) $z_1 = (x^3 - y^3) - (x - y) \cdot i,\ z_2 = \overline{1 + i}$;

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v) $z_1 = (x^3 + y^3) + (x + y) \cdot i$, $z_2 = \overline{\left(\frac{1+i}{1-i}\right)^{2014} + 1}$;

w) $z_1 = (x + y \cdot i) \cdot (1 - 2 \cdot i)$, $z_2 = \overline{-5 + 13 \cdot i} - 1$;

x) $z_1 = (x - y \cdot i) \cdot (-2 + i)$, $z_2 = \overline{-5 - 7 \cdot i} + 1$;

y) $z_1 = 5 \cdot \frac{x + y \cdot i}{3 + i}$, $z_2 = x \cdot (1 - i) - y$;

z) $z_1 = (-34) \cdot \frac{x - y \cdot i}{4 + i} + 7 \cdot x - 11 \cdot y \cdot i$, $z_2 = \overline{\left(\frac{1-i}{1+i}\right)^{2014}}$.

4. U Gaussovoj ravnini skicirajte sljedeće skupove:

a) $S = \{z \in \mathbb{C}: \operatorname{Re}(z) > 0\}$;

b) $S = \{z \in \mathbb{C}: \operatorname{Im}(z) < 0\}$;

c) $S = \{z \in \mathbb{C}: 2 \cdot \operatorname{Re}(z) \leq 0\}$;

d) $S = \{z \in \mathbb{C}: (-2) \cdot \operatorname{Im}(z) \leq 0\}$;

e) $S = \{z \in \mathbb{C}: \operatorname{Re}[(-2) \cdot z] \geq 0\}$;

f) $S = \{z \in \mathbb{C}: \operatorname{Im}(-z) \leq 0\}$;

g) $S = \{z \in \mathbb{C}: \operatorname{Im}\left(\overline{z}\right) < 0\}$;

h) $S = \{z \in \mathbb{C}: \operatorname{Re}\left(\overline{(-2) \cdot z}\right) > 4\}$;

i) $S = \{z \in \mathbb{C}: \operatorname{Re}(z) - \operatorname{Im}(z) > 0\}$;

j) $S = \{z \in \mathbb{C}: \operatorname{Re}(z) < \operatorname{Im}(z)\}$;

k) $S = \{z \in \mathbb{C}: \operatorname{Re}(2 \cdot z) < \operatorname{Im}(\overline{z})\}$;

l) $S = \{z \in \mathbb{C}: 2 \cdot \operatorname{Re}(\overline{z}) > \operatorname{Im}(-z)\}$;

m) $S = \{z \in \mathbb{C}: \operatorname{Re}\left[\left(-\frac{1}{2}\right) \cdot \overline{z}\right] + 2 \cdot \operatorname{Im}\left(\frac{1}{2} \cdot z\right) < 0\}$;

n) $S = \{z \in \mathbb{C}: \operatorname{Re}(z) + \operatorname{Im}(\overline{z}) > 2\}$;

o) $S = \{z \in \mathbb{C}: \operatorname{Re}(\overline{z}) - \operatorname{Im}(z) < 1\}$;

p) $S = \{z \in \mathbb{C}: z = -2 \cdot \overline{z}\}$;

q) $S = \{z \in \mathbb{C}: z = 4 \cdot \overline{z}\}$;

r) $S = \{z \in \mathbb{C}: 2 \cdot \overline{z} = (-2) \cdot z\}$;

s) $S = \{z \in \mathbb{C}: z + \overline{z} = \operatorname{Im}(-z)\}$;

t) $S = \{z \in \mathbb{C}: z - \overline{z} = \operatorname{Re}(-z)\}$;

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u) $S = \{z \in \mathbb{C}:\ \text{Re}\left(2 \cdot z - 3 \cdot \overline{z}\right) = \text{Im}\left(3 \cdot z + 2 \cdot \overline{z}\right)\}$;

v) $S = \{z \in \mathbb{C}:\ \text{Re}\left(z^2\right) \le 0\}$;

w) $S = \{z \in \mathbb{C}:\ \text{Re}\left(-z^2\right) > 0\}$;

x) $S = \{z \in \mathbb{C}:\ \overline{z} \cdot z - 3 \cdot \left[\text{Im}\left(\overline{z}\right)\right]^2 \ge 0\}$;

y) $S = \{z \in \mathbb{C}:\ 2 \cdot \left[\text{Re}\left(\overline{z}\right)\right]^2 - \overline{z} \cdot z \ge 0\}$;

z) $S = \{z \in \mathbb{C}:\ 4 \cdot \text{Re}(z) - [\text{Im}(z)]^2 \le 0\}$.

5. U Gaussovoj ravnini skicirajte sljedeće skupove:

a) $S = \{z \in \mathbb{C}: |z| = 2\}$;

b) $S = \left\{z \in \mathbb{C}: \left|\overline{z}\right| = 2\right\}$;

c) $S = \left\{z \in \mathbb{C}: z \cdot \overline{z} = 9\right\}$;

d) $S = \{z \in \mathbb{C}: |z + 2 \cdot i| = 2\}$;

e) $S = \{z \in \mathbb{C}: |z - i| = 3\}$;

f) $S = \{z \in \mathbb{C}: |z - 1 + i| = 2\}$;

g) $S = \{z \in \mathbb{C}: |z + 1 - i| = 3\}$;

h) $S = \{z \in \mathbb{C}: |z - 1 - i| = 1\}$;

i) $S = \{z \in \mathbb{C}: |z + 1 - i| < 3\}$;

j) $S = \{z \in \mathbb{C}: |z - 1 + i| \le 1\}$;

k) $S = \{z \in \mathbb{C}: |z - 2 - i| < 2\}$;

l) $S = \{z \in \mathbb{C}: |z - 2 + 2 \cdot i| \le 1\}$;

m) $S = \{z \in \mathbb{C}: |z + 2 + 3 \cdot i| > 1\}$;

n) $S = \{z \in \mathbb{C}: |z - 2 + 3 \cdot i| \ge 2\}$;

o) $S = \{z \in \mathbb{C}: |z - 3 - 2 \cdot i| > 3\}$;

p) $S = \{z \in \mathbb{C}: |z - 3 + 2 \cdot i| \ge 3\}$;

q) $S = \{z \in \mathbb{C}: \text{Re}(z) \cdot \text{Im}(z) \le 1\}$;

r) $S = \{z \in \mathbb{C}: 2 \cdot \text{Re}(z) \cdot \text{Im}(z) < 1\}$;

s) $S = \{z \in \mathbb{C}: \text{Re}(z) \cdot \text{Im}(z) > 2\}$;

t) $S = \{z \in \mathbb{C}: 2 \cdot \text{Re}(z) \cdot \text{Im}(z) \ge 3\}$;

u) $S = \{z \in \mathbb{C}:\ \text{Re}\left(\overline{z}\right) \cdot \text{Im}\left(2 \cdot z\right) < 4\}$;

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v) $S = \{z \in \mathbb{C}: \operatorname{Re}(z) \cdot \operatorname{Im}(3 \cdot \overline{z}) < -6\}$;

w) $S = \left\{z \in \mathbb{C} : \operatorname{Re}\left(\frac{1}{2} \cdot z\right) \cdot \operatorname{Im}\left[(-3) \cdot \overline{z}\right] > -3\right\}$;

x) $S = \{z \in \mathbb{C}: \operatorname{Re}(z^2) = -1\}$;

y) $S = \{z \in \mathbb{C}: [\operatorname{Im}(z^2)]^2 < 8 \cdot [\operatorname{Re}(z)]^3\}$;

z) $S = \left\{z \in \mathbb{C} : \left[\operatorname{Im}\left(\overline{z^2}\right)\right]^2 > 4 \cdot \left[\operatorname{Re}\left(\overline{z}\right)\right]^3\right\}$.

6. **a)** Neka je $f : \mathbb{C} \to \mathbb{C}$ funkcija definirana s $f(x) = a \cdot x + b$, gdje su $a, b \in \mathbb{R}$ konstante. Pokažite da za svaki $z \in \mathbb{C}$ vrijedi jednakost $f\left(\overline{z}\right) = \overline{f(z)}$.

b) Neka je $f : \mathbb{C} \to \mathbb{C}$ funkcija definirana s $f(x) = a \cdot x^2 + b \cdot x + c$, gdje su $a, b, c \in \mathbb{R}$ konstante. Pokažite da za svaki $z \in \mathbb{C}$ vrijedi jednakost $f\left(\overline{z}\right) = \overline{f(z)}$.

c) Vrijede li jednakosti iz prethodnih dvaju podzadataka ako pretpostavimo da su a, b, c kompleksne konstante? Obrazložite svoj odgovor.

7. Neka je $f : \mathbb{C} \to \mathbb{R}$ funkcija definirana s $f(x) = \operatorname{Re}(x) + \operatorname{Im}(x)$.

a) Pokažite da za sve $x, y \in \mathbb{C}$ vrijedi jednakost: $f(x + y) = f(x) + f(y)$.

b) Pokažite da za svaki $\alpha \in \mathbb{R}$ i za svaki $z \in \mathbb{C}$ vrijedi jednakost $f(\alpha \cdot z) = \alpha \cdot f(z)$.

c) Za svaki $z \in \mathbb{C}$ usporedite brojeve $f(z)$ i $|f(z)|$.

8. Neka je $f : \mathbb{C} \to \mathbb{R}$ funkcija definirana s $f(x) = \operatorname{Re}(x) - \operatorname{Im}(x)$.

a) Pokažite da za sve $x, y \in \mathbb{C}$ vrijedi jednakost: $f(x + y) = f(x) + f(y)$.

b) Pokažite da za svaki $\alpha \in \mathbb{R}$ i svaki $z \in \mathbb{C}$ vrijedi jednakost $f(\alpha \cdot z) = \alpha \cdot f(z)$.

c) Za svaki $z \in \mathbb{C}$ usporedite brojeve $f(z)$ i $|f(z)|$.

9. Neka je $f : \mathbb{R} \to \mathbb{C}$ funkcija definirana s $f(x) = (1 + i) \cdot x$.

a) Pokažite da za sve $x, y \in \mathbb{R}$ vrijedi jednakost: $f(x + y) = f(x) + f(y)$.

b) Pokažite da za sve $\alpha, x \in \mathbb{R}$ vrijedi jednakost $f(\alpha \cdot x) = \alpha \cdot f(x)$.

c) Za svaki $x \in \mathbb{R}$ usporedite brojeve $|f(x)|$ i $\left|f(-x)\right|$.

10. Neka je $f : \mathbb{R} \to \mathbb{C}$ funkcija definirana s $f(x) = \overline{(x - x \cdot i)^2}$

a) Pokažite da za sve $x, y \in \mathbb{R}$ vrijedi jednakost: $f(x + y) = f(x) + f(y)$.

b) Pokažite da za sve $\alpha, x \in \mathbb{R}$ vrijedi jednakost $f(\alpha \cdot x) = \alpha \cdot f(x)$.

c) Za svaki $x \in \mathbb{R}$ usporedite brojeve $|f(x)|$ i $\left|f(-x)\right|$.