OSNOVNE TEHNIKE DERIVIRANJA

1. Koristeći definiciju derivacije funkcije u točki izračunajte sljedeće granične vrijednosti:

a) $\lim\limits_{x\to 0}\dfrac{e^{2\cdot x}-1}{x}$;

b) $\lim\limits_{x\to 0}\dfrac{1-e^{x^2}}{x}$;

c) $\lim\limits_{x\to 0}\dfrac{e^{-x}-1}{2\cdot x}$

d) $\lim\limits_{x\to 0}\dfrac{\ln(2\cdot x+1)}{3\cdot x}$;

e) $\lim\limits_{x\to 0}\dfrac{\ln(x^2+1)}{4\cdot x}$;

f) $\lim\limits_{x\to 1}\dfrac{\ln^2 x}{1-x}$;

g) $\lim\limits_{x\to 0}\dfrac{e^{\sin x}-1}{5\cdot x}$;

h) $\lim\limits_{x\to 0}\dfrac{1-e^{\operatorname{tg} x}}{6\cdot x}$;

i) $\lim\limits_{x\to \frac{\pi}{2}}\dfrac{e^{\cos x}-1}{2\cdot x-\pi}$;

j) $\lim\limits_{x\to \pi}\dfrac{e^{\operatorname{ctg} x}-1}{\pi-x}$;

k) $\lim\limits_{x\to 0}\dfrac{\ln(e^x-1)}{x}$:

l) $\lim\limits_{x\to \frac{\pi}{2}}\dfrac{\ln(\sin x)}{\pi-2\cdot x}$;

m) $\lim\limits_{x\to 0}\dfrac{\ln(\cos x)}{2\cdot x}$;

n) $\lim\limits_{x\to \frac{\pi}{4}}\dfrac{\ln(\operatorname{tg} x)}{\pi-4\cdot x}$;

o) $\lim\limits_{x\to \frac{3\cdot\pi}{4}}\dfrac{\ln(-\operatorname{ctg} x)}{4\cdot x-3\cdot\pi}$;

p) $\lim\limits_{x\to \frac{\pi}{2}}\dfrac{e^{\cos x}+\ln(\sin x)-1}{2\cdot x-\pi}$;

q) $\lim\limits_{x\to \frac{3\cdot\pi}{2}}\dfrac{\sin(\cos x)}{2\cdot x-3\cdot\pi}$;

r) $\lim\limits_{x\to 0}\dfrac{\cos(\sin x)-1}{x}$;

OSNOVNE TEHNIKE DERIVIRANJA

s) $\lim\limits_{x\to\pi}\dfrac{\operatorname{tg}(\sin x)}{x-\pi}$;

t) $\lim\limits_{x\to\pi}\dfrac{\operatorname{ctg}\left(\dfrac{\pi}{2}\cdot\cos x\right)}{\pi-x}$;

u) $\lim\limits_{x\to 0}\dfrac{2\cdot\arcsin(x-1)+\pi}{4\cdot x}$;

v) $\lim\limits_{x\to 0}\dfrac{2\cdot\arccos x-\pi}{3\cdot x}$;

w) $\lim\limits_{x\to 0}\dfrac{4\cdot\operatorname{arctg}(x+1)-\pi}{2\cdot x}$;

x) $\lim\limits_{x\to 2}\dfrac{4\cdot\operatorname{arcctg}(1-x)+3\cdot\pi}{x-2}$;

y) $\lim\limits_{x\to 0}\dfrac{1-e^{\arcsin x}}{x}$;

z) $\lim\limits_{x\to 0}\dfrac{\ln\left(\dfrac{2}{\pi}\cdot\arccos x\right)}{2\cdot x}$.

2. Primjenom pravila za deriviranje složene funkcije odredite prvu derivaciju sljedećih funkcija i pojednostavnite dobiveni izraz što više možete:

a) $f(x)=(x^2-1)^{2014}$;

b) $f(x)=\left(\dfrac{x-1}{2}\right)^{2015}$;

c) $f(x)=(x^2+x+1)^{2013}$;

d) $f(x)=(\sin x+\cos x)^{2011}$;

e) $f(x)=(2\cdot x-3\cdot e^x)^{2010}$;

f) $f(x)=\sqrt{x^2-3\cdot x+2}$;

g) $f(x)=\sqrt[3]{(a\cdot x+b)^2}$;

h) $f(x)=\dfrac{1}{\sqrt{a\cdot x^2+b\cdot x+c}}$;

i) $f(x)=\sqrt{e^x-x}$;

j) $f(x)=\sqrt{x+\ln x}$;

k) $f(x)=\sqrt[3]{\left(\sqrt{x^2+1}+\sqrt{x^2-1}\right)^2}$;

l) $f(x)=e^{3\cdot x}-\ln^3 x$;

m) $f(x)=\sqrt{e^{1-x}}+(\ln x)\cdot\sqrt{\ln x}$;

n) $f(x)=\sin(x^2)+\cos\sqrt{x}$;

OSNOVNE TEHNIKE DERIVIRANJA

o) $f(x)=\operatorname{tg}\sqrt{x}-\operatorname{ctg}\sqrt{x+1}$;

p) $f(x)=2\cdot\sin\frac{x+1}{2}-3\cdot\cos\frac{x-1}{3}+4\cdot\operatorname{tg}\frac{x+2}{4}-5\cdot\operatorname{ctg}\frac{x-2}{5}$;

q) $f(x)=\ln(\cos x)+\ln(\sin x)$;

r) $f(x)=e^{\sin x}+\sin(e^x)$;

s) $f(x)=\arcsin\sqrt{x+1}-\arccos\sqrt{x-1}$;

t) $f(x)=\operatorname{arctg}\sqrt{x+1}+\operatorname{arcctg}\sqrt{x-1}$;

u) $f(x)=\operatorname{arctg}\sqrt{\ln x}-\arcsin\sqrt{e^x+1}$;

v) $f(x)=\arcsin^2\left[\sqrt[3]{(x-1)^2}\right]+\arccos^2 x^2-\operatorname{arctg}^3\frac{1}{x\cdot\sqrt{x}}-\operatorname{arcctg}^3(x\cdot\sqrt{x})$;

w) $f(x)=e^{\arcsin x}+\arcsin(e^x)$;

x) $f(x)=e^{-\arccos(x^2)}-\arccos\left(e^{-x^2}\right)$;

y) $f(x)=\ln(\ln x)-\ln^2 x$;

z) $f(x)=e^x\cdot\cos\sqrt{e^{1-x}}-x\cdot\sin\sqrt{\ln x}$.

3. Odredite prvu derivaciju sljedećih funkcija i pojednostavnite dobiveni izraz što više možete (a, $b\in\mathbb{R}$ su konstante):

a) $f(x)=\frac{1}{6}\cdot\ln(x^2-x+1)+\frac{\sqrt{3}}{3}\cdot\operatorname{arctg}\left(\frac{2\cdot\sqrt{3}\cdot x-\sqrt{3}}{3}\right)+\frac{1}{3}\cdot\ln(x+1)$;

b) $f(x)=\frac{1}{54}\cdot\ln(x^2+3\cdot x+9)+\frac{\sqrt{3}}{27}\cdot\operatorname{arctg}\left(\frac{2\cdot\sqrt{3}\cdot x-\sqrt{3}}{9}\right)-\frac{1}{27}\cdot\ln(x-3)$;

c) $f(x)=\frac{1}{6}\cdot\ln(x^2-x+1)-\frac{1}{3}\cdot\ln(x+1)+\frac{1}{3}\cdot\sqrt{3}\cdot\operatorname{arctg}\left(\frac{2\cdot x-1}{\sqrt{3}}\right)$;

d) $f(x)=\operatorname{arcsh}\left(\frac{2\cdot x+1}{3}\right)$;

e) $f(x)=\operatorname{arcsh}\left(\frac{2\cdot\sqrt{3}\cdot x+\sqrt{3}}{3}\right)$;

f) $f(x)=\ln\left(2\cdot x+\sqrt{4\cdot x^2-4}\right)$;

g) $f(x)=\frac{1}{2}\cdot\ln(x^2+1)+\operatorname{arctg} x$;

h) $f(x)=1-2\cdot\operatorname{arcth}\left(\sqrt{e^x+1}\right)$;

i) $f(x)=2\cdot\operatorname{arctg}\left(\sqrt{e^x-1}\right)-1$;

j) $f(x)=\sqrt{1-x^2}+x\cdot\arcsin x$;

OSNOVNE TEHNIKE DERIVIRANJA

k) $f(x) = x \cdot \arccos x - \sqrt{1-x^2} + 1$;

l) $f(x) = x \cdot \operatorname{arctg} x - \frac{1}{2} \cdot \ln(x^2+1)$;

m) $f(x) = x \cdot \operatorname{arcctg} x + \frac{1}{2} \cdot \ln(x^2+1)$;

n) $f(x) = x \cdot \operatorname{arcsh} x - \sqrt{x^2+1}$;

o) $f(x) = \frac{1}{\cos^2 x} + \ln(\cos x) + 1$;

p) $f(x) = \frac{1}{\sin^2 x} - \frac{1}{4 \cdot \sin^4 x} + \ln(\sin x) - 2014$;

q) $f(x) = \frac{1}{2} \cdot (x^2+1) \cdot \operatorname{arctg} x - \frac{1}{2} \cdot x - 1$;

r) $f(x) = \frac{1}{8} \cdot \{2 \cdot x \cdot [x - \sin(2 \cdot x)] - \cos(2 \cdot x)\}$;

s) $f(x) = \frac{1}{2} \cdot \left[\ln\left(\sqrt{\cos x} - 1\right) - \ln\left(\sqrt{\cos x} + 1\right) - 2 \cdot \operatorname{arctg}\left(\sqrt{\cos x}\right) \right]$;

t) $f(x) = \frac{1}{3} \cdot \ln(x^3+1) - \frac{1}{6} \cdot \ln(x^2 - x + 1) + \frac{1}{3} \cdot \ln(x+1) - \frac{\sqrt{3}}{3} \cdot \operatorname{arctg}\left(\frac{2 \cdot \sqrt{3} \cdot x - \sqrt{3}}{3} \right)$;

u) $f(x) = \frac{1}{4} \cdot \left[\ln(x^4+1) + \sqrt{2} \cdot \operatorname{arcth}\left(\frac{\sqrt{2} \cdot x}{x^2+1} \right) - \sqrt{2} \cdot \operatorname{arctg}\left(1 - \sqrt{2} \cdot x\right) + \sqrt{2} \cdot \operatorname{arctg}\left(\sqrt{2} \cdot x + 1\right) \right]$;

v) $f(x) = \frac{1}{2} \cdot \left[x \cdot \sqrt{x^2+16} + 8 \cdot \operatorname{arcsh}\left(\frac{x}{4} \right) \right]$;

w) $f(x) = \frac{1}{2} \cdot x \cdot \sqrt{x^2-9} - \frac{9}{2} \cdot \ln\left[2 \cdot \left(\sqrt{x^2-9} + x \right) \right]$;

x) $f(x) = \frac{2}{\sqrt{a^2-b^2}} \cdot \operatorname{arctg}\left[\frac{a \cdot \operatorname{tg}\left(\frac{x}{2}\right) + b}{\sqrt{a^2-b^2}} \right]$;

y) $f(x) = 2010 - \frac{2}{\sqrt{b^2-a^2}} \cdot \operatorname{arcth}\left[\frac{(a+b) \cdot \operatorname{tg}\frac{x}{2}}{\sqrt{b^2-a^2}} \right]$;

z) $f(x) = \frac{1}{2} \cdot \left[\arcsin\sqrt{x} + 2 \cdot x \cdot \arccos\sqrt{x} - \sqrt{x - x^2} \right]$.

4. Primjenom pravila za deriviranje implicitno zadane funkcije odredite prve derivacije sljedećih implicitno zadanih funkcija i pojednostavnite dobivene izraze što više možete (u svim zadacima su $a, b, p, q \in \mathbb{R}$ konstante):

a) $(x-p)^2 + (y-q)^2 = r^2$;
b) $b^2 \cdot (x+p)^2 + a^2 \cdot (y+q)^2 = a^2 \cdot b^2$;

OSNOVNE TEHNIKE DERIVIRANJA

c) $b^2 \cdot (x - 2 \cdot p)^2 - a^2 \cdot (y - 2 \cdot q)^2 = a^2 \cdot b^2$;
d) $y^2 = 2 \cdot p \cdot (x + a)$;
e) $x^3 + y^3 = a^3$;
f) $\sqrt{x} + \sqrt{y} = \sqrt{a}$;
g) $x \cdot y + \sin y = 0$;
h) $\cos\sqrt{y} = \frac{y}{x}$;
i) $\operatorname{arctg}(x + y) = x$;
j) $x^y - y^x = 1$;
k) $\ln y = 2014 - \frac{x}{y}$;
l) $x \cdot e^y - y \cdot e^x = 2015$;
m) $(x - y)^3 = 64 \cdot (y - x)$;
n) $x \cdot y^3 - 2 \cdot y = 2014$;
o) $y^2 - x = \ln\frac{y}{x}$;
p) $y + \cos y = x^2$;
q) $x \cdot \sqrt{1 + \cos y} + (\cos x) \cdot \sqrt{1 + x} = 2015$;
r) $x \cdot e^{x+y} + y \cdot e^{x-y} = 1$;
s) $\sqrt{x^2 + y^2} = 2014 \cdot \operatorname{arctg}\frac{y}{x}$;
t) $x^3 + x^2 \cdot y + x \cdot y^2 + y^3 = a^3$;
u) $(x + y) \cdot \ln(x + y) + y = 1$;
v) $a \cdot \sin^2(y - x) = b$;
w) $\operatorname{ctg} y = \frac{y}{x}$;
x) $x \cdot \arcsin\sqrt{y} + y \cdot \arcsin\sqrt{x} = \sqrt{x \cdot y}$;
y) $x \cdot \operatorname{arsh} y + y \cdot \operatorname{arch} x = 2015$;
z) $x \cdot \operatorname{ch}(x + y) + (x - y) \cdot \operatorname{sh} x = e^x$.

5. Koristeći logaritamsko deriviranje odredite prve derivacije sljedećih funkcija i pojednostavnite dobivene izraze što više možete:

a) $f(x) = (x-1)^2 \cdot (x+2)^6$;
b) $f(x) = (x+1)^6 \cdot (x-2)^2$;
c) $f(x) = (x+2) \cdot (2 \cdot x+3) \cdot (3 \cdot x+4) \cdot (4 \cdot x+5)$;
d) $f(x) = (x-2)^3 \cdot (4 \cdot x-5)^6 \cdot (7 \cdot x-8)^9$;
e) $f(x) = \frac{(x-1)^3}{(x+4)^7}$;

OSNOVNE TEHNIKE DERIVIRANJA

f) $f(x)=\dfrac{(x+2)^3\cdot(x-1)^5}{(x+1)^4}$;

g) $f(x)=\dfrac{(x-2)^3}{(x-1)^7\cdot(x-3)^5}$;

h) $f(x)=\dfrac{(x-7)^6\cdot(x+5)^4}{(x-3)^2\cdot(x+1)}$;

i) $f(x)=\dfrac{(x-1)^2\cdot(x+3)^4}{(x-5)^6\cdot(x-7)^8}$;

j) $f(x)=\sqrt{\dfrac{x\cdot(x-1)}{x+1}}$

k) $f(x)=(x-1)\cdot\sqrt{\dfrac{x-2}{x+3}}$;

l) $f(x)=(x-2)\cdot\sqrt[3]{\left(\dfrac{x+4}{x-5}\right)^2}$;

m) $f(x)=\dfrac{x\cdot(x+1)}{\sqrt{(x-1)\cdot(x+2)}}$;

n) $f(x)=\dfrac{\sqrt[3]{\left[(x-1)\cdot(x+2)\right]^2}}{(x+1)^3\cdot(x-2)^4}$;

o) $f(x)=\dfrac{\sqrt{x-1}}{\sqrt[3]{x-2}}$

p) $f(x)=\dfrac{\sqrt[3]{\left[(x+4)\cdot(x^2-x)\right]^2}}{\sqrt{(x^2+x)\cdot(x-4)}}$;

q) $f(x)=x^{\sqrt[3]{x}}$;

r) $f(x)=x^{x^3}$;

s) $f(x)=x^{x^{\sqrt{x}}}$;

t) $f(x)=x^{\cos x}$;

u) $f(x)=(\cos x)^x$;

v) $f(x)=\left(1-\dfrac{1}{x}\right)^x$;

w) $f(x)=\left(1-\dfrac{1}{x+1}\right)^{x+1}$;

x) $f(x)=(1+x)^{\frac{1}{x}}$;

y) $f(x)=\sqrt{(x-1)^{\frac{1}{x}}}$;

z) $f(x)=(\ln x)^{\cos x}$.

OSNOVNE TEHNIKE DERIVIRANJA

6. Koristeći pravilo za deriviranje parametarski zadane funkcije odredite prve derivacije sljedećih funkcija i pojednostavnite dobivene izraze što više možete (a, $b \in \mathbb{R}$ su konstante):

a) $\begin{cases} x = t, \\ y = 2 \cdot t; \end{cases}$

b) $\begin{cases} x = 1 - t, \\ y = 4 \cdot t + 3; \end{cases}$

c) $\begin{cases} x = t^2, \\ y = 1 - 2 \cdot t; \end{cases}$

d) $\begin{cases} x = 1 + t + t^2, \\ y = 1 - 2 \cdot t - 3 \cdot t^2; \end{cases}$

e) $\begin{cases} x = -\sqrt[3]{t}, \\ y = \sqrt{t}; \end{cases}$

f) $\begin{cases} x = \sqrt[3]{t^2 + 1}, \\ y = -\sqrt{t + 1}; \end{cases}$

g) $\begin{cases} x = \sqrt[3]{t^2 + 1}, \\ y = -\sqrt{t + 1}; \end{cases}$

h) $\begin{cases} x = \sqrt{t^2 + a^2}, \\ y = \dfrac{t - a}{\sqrt{t^2 + a^2}}; \end{cases}$

i) $\begin{cases} x = \dfrac{1}{t + a}, \\ y = \dfrac{t^2}{(t + a)^2}; \end{cases}$

j) $\begin{cases} x = \dfrac{a \cdot t}{1 + t^3}, \\ y = \dfrac{a \cdot t^2}{1 + t^3}; \end{cases}$

k) $\begin{cases} x = \dfrac{2 \cdot a \cdot t}{1 + t^2}, \\ y = \dfrac{a \cdot (1 - t^2)}{1 + t^2}; \end{cases}$

l) $\begin{cases} x = a \cdot \dfrac{t^2}{1 + t^2}, \\ y = a \cdot \dfrac{t^3}{1 + t^2}; \end{cases}$

OSNOVNE TEHNIKE DERIVIRANJA

m) $\begin{cases} x = a \cdot \cos t, \\ y = b \cdot \sin t; \end{cases}$

n) $\begin{cases} x = a \cdot \cos^2 t, \\ y = b \cdot \sin^2 t; \end{cases}$

o) $\begin{cases} x = a \cdot \operatorname{tg} t, \\ y = b \cdot \cos^2 t; \end{cases}$

p) $\begin{cases} x = a \cdot \operatorname{ctg} t, \\ y = -b \cdot \sin^2 t; \end{cases}$

q) $\begin{cases} x = a \cdot (\cos t + t \cdot \sin t), \\ y = b \cdot (\sin t - t \cdot \cos t); \end{cases}$

r) $\begin{cases} x = a \cdot (t - \sin t), \\ y = a \cdot (1 - \cos t); \end{cases}$

s) $\begin{cases} x = \dfrac{\cos^3 t}{\sqrt{\cos(2 \cdot t)}}, \\ y = \dfrac{\sin^3 t}{\sqrt{\cos(2 \cdot t)}}; \end{cases}$

t) $\begin{cases} x = \arccos \dfrac{1}{\sqrt{t^2 + a^2}}, \\ y = \arcsin \dfrac{t}{\sqrt{t^2 + a^2}}; \end{cases}$

u) $\begin{cases} x = e^{2 \cdot t}, \\ y = e^{3 \cdot t}; \end{cases}$

v) $\begin{cases} x = e^{1-t}, \\ y = e^{t+1}; \end{cases}$

w) $\begin{cases} x = a \cdot \operatorname{ch} t, \\ y = a \cdot \operatorname{sh} t; \end{cases}$

x) $\begin{cases} x = a \cdot \operatorname{ch}^2 t, \\ y = a \cdot \operatorname{sh}^2 t; \end{cases}$

y) $\begin{cases} x = a \cdot \operatorname{th}^2 t, \\ y = a \cdot \operatorname{cth}^2 t; \end{cases}$

z) $\begin{cases} x = \operatorname{arth} \sqrt{t}, \\ y = \operatorname{arcth} \sqrt{t}. \end{cases}$

7. Koristeći se pravilom za deriviranje inverza funkcije izvedite derivacije svih četiriju area funkcija.