

**DETERMINANTE REDA 2 I 3. REGULARNE MATRICE.**

1. Riješite jednađbu $D = 0$ ako je:

a) $D = \begin{vmatrix} a & -1 \\ 1 & 1 \end{vmatrix};$

b) $D = \begin{vmatrix} 1 & -a \\ -1 & 1 \end{vmatrix};$

c) $D = \begin{vmatrix} 2 & 3 \\ 4 & a \end{vmatrix};$

d) $D = \begin{vmatrix} 2 & -4 \\ a & 6 \end{vmatrix};$

e) $D = \begin{vmatrix} a+1 & a+2 \\ 2 & 3 \end{vmatrix};$

f) $D = \begin{vmatrix} a+1 & 3 \\ a-1 & 2 \end{vmatrix};$

g) $D = \begin{vmatrix} a & a-1 \\ a-1 & a+1 \end{vmatrix};$

h) $D = \begin{vmatrix} a+1 & a+2 \\ a+3 & a+4 \end{vmatrix};$

i) $D = \begin{vmatrix} a+1 & -1 \\ 2 & a+4 \end{vmatrix};$

j) $D = \begin{vmatrix} a-1 & -1 \\ -2 & a-2 \end{vmatrix};$

k) $D = \begin{vmatrix} a+2 & -1 \\ -4 & a-1 \end{vmatrix};$

l) $D = \begin{vmatrix} 1-a & -4 \\ -2 & a+1 \end{vmatrix};$

m) $D = \begin{vmatrix} 2-a & 2 \\ 8 & a-2 \end{vmatrix};$

n) $D = \begin{vmatrix} a+3 & -9 \\ -4 & a+3 \end{vmatrix};$

o) $D = \begin{vmatrix} a+3 & -4 \\ -4 & a-3 \end{vmatrix};$

p) $D = \begin{vmatrix} a & 3 & 2 \\ 0 & 1 & 2 \\ 0 & 0 & 2 \end{vmatrix};$

**DETERMINANTE REDA 2 I 3. REGULARNE MATRICE.**

$$\mathbf{q)} \quad D = \begin{vmatrix} 1 & a+1 & 2 \\ 1 & 0 & 2 \\ 3 & 0 & 4 \end{vmatrix};$$

$$\mathbf{r)} \quad D = \begin{vmatrix} 1 & -1 & a-1 \\ 1 & 1 & 0 \\ 3 & 2 & 0 \end{vmatrix};$$

$$\mathbf{s)} \quad D = \begin{vmatrix} a & -a & a \\ 1 & 2 & 3 \\ 4 & 5 & 6 \end{vmatrix};$$

$$\mathbf{t)} \quad D = \begin{vmatrix} a+1 & 1 & -1 \\ a-1 & -1 & 0 \\ 1-a & 1 & 1 \end{vmatrix};$$

$$\mathbf{u)} \quad D = \begin{vmatrix} 1 & 3 & 2 \\ 0 & a & 2 \\ 0 & 2 & 1 \end{vmatrix};$$

$$\mathbf{v)} \quad D = \begin{vmatrix} 1 & 0 & 0 \\ -\pi & -a & 1 \\ e^2 & 4 & 2 \end{vmatrix};$$

$$\mathbf{w)} \quad D = \begin{vmatrix} -1 & 0 & 3 \\ e & -1 & \pi \\ a & 0 & 6 \end{vmatrix};$$

$$\mathbf{x)} \quad D = \begin{vmatrix} 1 & -e & a \\ 0 & -2 & 0 \\ 4 & \pi^2 & -8 \end{vmatrix};$$

$$\mathbf{y)} \quad D = \begin{vmatrix} 1 & 1 & 0 \\ a^2 & 9 & 0 \\ \sqrt{e} & \sqrt[3]{\pi} & 5 \end{vmatrix};$$

$$\mathbf{z)} \quad D = \begin{vmatrix} a^2 & -1 & -1 \\ \sqrt[3]{e} & a^3 & 64 \\ \sqrt{\pi} & 0 & 0 \end{vmatrix}.$$

2. Neka je $a \in \mathbb{R}$ parametar. Pokažite da je vrijednost determinante

$$D = \begin{vmatrix} 1-a & a \\ -2 \cdot a & a+1 \end{vmatrix}$$

strogopozitivna neovisno o vrijednosti parametra a .

DETERMINANTE REDA 2 I 3. REGULARNE MATRICE.

3. Neka je $a \in \mathbb{R}$ parametar. Pokažite da vrijednost determinante

$$D = \begin{vmatrix} a & a+1 & a+2 \\ a+3 & a+4 & a+5 \\ a+6 & a+7 & a+8 \end{vmatrix}$$

ne ovisi o vrijednosti parametra a .

4. Riješite jednadžbu $D = 0$ ako je

$$D = \begin{vmatrix} \cos a & \sin a \\ \sin a & \cos a \end{vmatrix}.$$

5. Neka je $a \in \mathbb{R}$ parametar. Pokažite da je vrijednost determinante

$$D = \begin{vmatrix} \cos a & 0 & -1 \\ \sin a & -1 & 0 \\ 0 & 1 & 1 \end{vmatrix}$$

jednaka nuli ako i samo ako je $a \in \left\{ \frac{3 \cdot \pi}{4} \cdot \left(\frac{4}{3} \cdot k + 1 \right) : k \in \mathbf{Z} \right\}$.

6. Izravno iz definicije inverza provjerite je li matrica B inverz matrice A ako su:

a) $A = [2016]$ i $B = \left[\frac{1}{2016} \right]$;

b) $A = [-2015]$ i $B = \left[\frac{1}{2015} \right]$;

c) $A = \begin{bmatrix} 4 & 9 \\ 3 & 7 \end{bmatrix}$ i $B = \begin{bmatrix} 7 & -9 \\ 3 & 4 \end{bmatrix}$;

d) $A = \begin{bmatrix} 1 & 3 \\ 2 & 7 \end{bmatrix}$ i $B = \begin{bmatrix} 7 & -3 \\ -2 & 1 \end{bmatrix}$;

e) $A = \begin{bmatrix} 2014 & 1 \\ 2015 & 1 \end{bmatrix}$ i $B = \begin{bmatrix} -1 & 1 \\ 2015 & 2014 \end{bmatrix}$;

f) $A = \begin{bmatrix} 7 & 11 \\ 5 & 8 \end{bmatrix}$ i $B = \begin{bmatrix} 8 & -11 \\ -5 & 7 \end{bmatrix}$;

g) $A = \begin{bmatrix} 9 & 4 \\ 25 & 11 \end{bmatrix}$ i $B = \begin{bmatrix} -11 & 25 \\ 4 & -9 \end{bmatrix}$

h) $A = \begin{bmatrix} 2 & 2 & 8 \\ 0 & 1 & 3 \\ 3 & 0 & 2 \end{bmatrix}$ i $B = \begin{bmatrix} -1 & 2 & 1 \\ -4.5 & 10 & 3 \\ 1.5 & -3 & 1 \end{bmatrix}$;

**DETERMINANTE REDA 2 I 3. REGULARNE MATRICE.**

$$\text{i) } A = \begin{bmatrix} 0 & 2 & 1 \\ -1 & -1 & 0 \\ 1 & 0 & -1 \end{bmatrix} \text{ i } B = \begin{bmatrix} -1 & -2 & -1 \\ 1 & 1 & 1 \\ -1 & -2 & -2 \end{bmatrix};$$

$$\text{j) } A = \begin{bmatrix} -2 & 1 & 3 \\ 5 & 0 & 11 \\ 4 & 0 & 9 \end{bmatrix} \text{ i } B = \begin{bmatrix} 0 & 9 & -11 \\ 1 & 30 & -37 \\ 0 & -4 & 5 \end{bmatrix};$$

$$\text{k) } A = \begin{bmatrix} -2 & 1 & 3 \\ 5 & 0 & 11 \\ 4 & 0 & 9 \end{bmatrix} \text{ i } B = \begin{bmatrix} 0 & 9 & -11 \\ 1 & 30 & -37 \\ 0 & -4 & 5 \end{bmatrix};$$

$$\text{l) } A = \begin{bmatrix} 7 & 8 & 5 \\ 1 & 0 & 0 \\ 9 & 3 & 2 \end{bmatrix} \text{ i } B = \begin{bmatrix} 0 & 1 & 0 \\ 2 & 31 & -5 \\ -3 & -51 & 8 \end{bmatrix}.$$

7. Matrica B je inverz matrice A . Izrazite pomoću matrice B inverz matrice:

- a) $2 \cdot A$;
- b) $3 \cdot A$;
- c) $(-2015) \cdot A$;
- d) $\frac{1}{3} \cdot A$;
- e) $\left(-\frac{2}{3}\right) \cdot A$;
- f) $\pi \cdot A$;
- g) $e \cdot A$;
- h) $\sqrt{3} \cdot A$;
- i) A^2 ;
- j) A^3 .

8. Odredite matricu A ako je:

- a) $A^{-1} = \begin{bmatrix} 1 & 0 \\ -4 & 1 \end{bmatrix}$;
- b) $A^{-1} = \begin{bmatrix} 8 & -7 \\ -9 & 8 \end{bmatrix}$;
- c) $2 \cdot A^{-1} = \begin{bmatrix} -7 & 6 \\ 5 & -4 \end{bmatrix}$;
- d) $3 \cdot A^{-1} = \begin{bmatrix} 6 & -19 \\ -3 & 10 \end{bmatrix}$;
- e) $4 \cdot A^{-1} = \begin{bmatrix} 2 \cdot \sqrt{3} & -1 \\ -1 & \frac{1}{2} \cdot \sqrt{3} \end{bmatrix}$;

DETERMINANTE REDA 2 I 3. REGULARNE MATRICE.

$$\text{f) } A^{-1} = \begin{bmatrix} 1 & 13 & -12 \\ 0 & 5 & -4 \\ 0 & -6 & 5 \end{bmatrix};$$

$$\text{g) } A^{-1} = \begin{bmatrix} 0 & 7 & -5 \\ 0 & -11 & 8 \\ 1 & 3 & -3 \end{bmatrix};$$

$$\text{h) } 4 \cdot A^{-1} = \begin{bmatrix} 0 & 1 & -2 \\ -12 & 8 & 4 \\ 8 & -5 & -2 \end{bmatrix};$$

$$\text{i) } 19 \cdot A^{-1} = \begin{bmatrix} -7 & 8 & 5 \\ 4 & 10 & -11 \\ 1 & -7 & 2 \end{bmatrix}$$

$$\text{j) } 21 \cdot A^{-1} = \begin{bmatrix} 14 & 14 & 7 \\ 22 & 19 & 5 \\ 9 & 3 & 3 \end{bmatrix};$$

$$\text{k) } (A^{-1})^T = \begin{bmatrix} 8 & -19 \\ -5 & 12 \end{bmatrix};$$

$$\text{l) } (A^{-1})^T = \begin{bmatrix} 5 & -8 \\ -8 & 13 \end{bmatrix};$$

$$\text{m) } (2 \cdot A^{-1})^T = \begin{bmatrix} 5 & 2 \\ 9 & 4 \end{bmatrix};$$

$$\text{n) } (6 \cdot A^{-1})^T = \begin{bmatrix} 4 & 10 \\ 5 & 14 \end{bmatrix};$$

$$\text{o) } (A^{-1})^T = \begin{bmatrix} 0 & 4 & -3 \\ 0 & 5 & -4 \\ 1 & 27 & -22 \end{bmatrix};$$

$$\text{p) } (A^{-1})^T = \begin{bmatrix} 8 & 83 & -3 \\ 0 & 1 & 0 \\ -5 & -53 & 2 \end{bmatrix};$$

$$\text{q) } (5 \cdot A^{-1})^T = \begin{bmatrix} 5 & -5 & -5 \\ 2 & -4 & -1 \\ -2 & -1 & -1 \end{bmatrix};$$

$$\text{r) } (10 \cdot A^{-1})^T = \begin{bmatrix} 7 & 44 & -1 \\ -4 & -28 & 2 \\ -3 & -26 & -1 \end{bmatrix}.$$

DETERMINANTE REDA 2 I 3. REGULARNE MATRICE.

9. Ispitajte jesu li sljedeće matrice regularne i, ako jesu, odredite njihov inverz:

a) $A = \begin{bmatrix} 1 & 2016 \\ 1 & 2015 \end{bmatrix};$

b) $A = \begin{bmatrix} 1 & 4 \\ -2 & -7 \end{bmatrix};$

c) $A = \begin{bmatrix} 0 & \ln 2017 \\ 0 & \ln 2016 \end{bmatrix};$

d) $A = \begin{bmatrix} 11 & -14 \\ -7 & 9 \end{bmatrix};$

e) $A = \begin{bmatrix} 11 & 6 \\ 9 & 5 \end{bmatrix};$

f) $A = \begin{bmatrix} 1 & 0 & 1 \\ -1 & 1 & 2 \\ 0 & -1 & -1 \end{bmatrix};$

g) $A = \begin{bmatrix} 3 & 0 & 2 \\ 1 & 2 & 3 \\ 0 & 2 & 2 \end{bmatrix};$

h) $A = \begin{bmatrix} 1 & -9 & 6 \\ 0 & 7 & -4 \\ 0 & -12 & 7 \end{bmatrix};$

i) $A = \begin{bmatrix} 0 & 6 & 5 \\ 0 & 13 & 11 \\ 1 & 69 & 59 \end{bmatrix};$

j) $A = \begin{bmatrix} 0 & -7 & 19 \\ 1 & -114 & 308 \\ 0 & -3 & 8 \end{bmatrix}.$

10. Neka je A dijagonalna matrica reda 3 čiji su svi elementi definirani pravilom:

$$a_{ii} = i, \text{ za svaki } i = 1, 2, 3.$$

Izračunajte $(6 \cdot A^{-1})^T$.

11. Neka je A gornja trokutasta matrica reda 3 čiji su svi elementi definirani pravilom:

$$a_{ij} = i + j, \text{ za svaki } j = 1, 2, 3 \text{ i svaki dopustivi } i \geq j.$$

Izračunajte $(48 \cdot A^{-1})^T$.

DETERMINANTE REDA 2 I 3. REGULARNE MATRICE.

12. Neka je A donja trokutasta matrica reda 3 definirana s

$$a_{ij} = 2 \cdot i - j, \text{ za svaki } j = 1, 2, 3 \text{ i svaki dopustivi } i \leq j.$$

Izračunajte $(6 \cdot A^{-1})^T$.

13. Pokažite da su sljedeće matrice regularne za svaki $x \in \mathbb{R}$ i odredite njihov inverz:

a) $A = \begin{bmatrix} 1 & x^2 \\ 0 & -1 \end{bmatrix};$

b) $A = \begin{bmatrix} x+1 & x+2 \\ x & x+1 \end{bmatrix};$

c) $A = \begin{bmatrix} x+1 & -2 \\ 1 & x-1 \end{bmatrix};$

d) $A = \begin{bmatrix} \cos(2 \cdot x) & \sin(-2 \cdot x) \\ \sin(2 \cdot x) & \cos(-2 \cdot x) \end{bmatrix};$

e) $A = \begin{bmatrix} e^x & e^{2 \cdot x} \\ 0 & e^{-x} \end{bmatrix};$

f) $A = \begin{bmatrix} 2-x & \sqrt{3}-x \\ x+\sqrt{3} & x+2 \end{bmatrix};$

g) $A = \begin{bmatrix} -1 & 1-x & 1+x \\ 0 & -1 & x \\ 0 & 0 & 1 \end{bmatrix};$

h) $A = \begin{bmatrix} \frac{1}{2} & 0 & 0 \\ x^2 & -8 & 0 \\ x & \frac{1}{2} \cdot x & \frac{1}{4} \end{bmatrix};$

i) $A = \begin{bmatrix} x^2+1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \frac{1}{x^2+1} \end{bmatrix};$

j) $A = \begin{bmatrix} 1 & x & -1 \\ 0 & 1 & 0 \\ -2 & x^2 & 3 \end{bmatrix};$

k) $A = \begin{bmatrix} -1 & 2 \cdot x^2 & 0 \\ 0 & 1 & 0 \\ x & 3 \cdot x^3 & 1 \end{bmatrix}.$

DETERMINANTE REDA 2 I 3. REGULARNE MATRICE.

14. Odredite sve vrijednosti $x \in \mathbb{R}$ za koje su sljedeće matrice regularne:

- a) $A = [x]$;
- b) $A = \begin{bmatrix} x+1 & 3 \\ 4 & 2 \end{bmatrix}$;
- c) $A = \begin{bmatrix} 1-x & -4 \\ 5 & 8 \end{bmatrix}$;
- d) $A = \begin{bmatrix} 1-x & 5 \\ 7 & x+1 \end{bmatrix}$;
- e) $A = \begin{bmatrix} x-2 & 10 \\ -3 & 1-x \end{bmatrix}$;
- f) $A = \begin{bmatrix} x+2 & 2x \\ 4-x & 3-x \end{bmatrix}$;
- g) $A = \begin{bmatrix} x & 1 & 0 \\ 1 & x & 0 \\ 0 & 1 & x \end{bmatrix}$;
- h) $A = \begin{bmatrix} -x & -1 & 1 \\ 2 & 1 & x+1 \\ x-1 & 0 & -1 \end{bmatrix}$;
- i) $A = \begin{bmatrix} 2014+x & 2015+x & 2016+x \\ 2017-x & 2018-x & 2019-x \\ 1 & 2 & 12 \end{bmatrix}$;
- j) $A = \begin{bmatrix} 2015-x & 2016-x & 2017-x \\ 2018+x & 2019+x & 2020+x \\ 2022 & 2023 & 2024 \end{bmatrix}$;
- k) $A = \begin{bmatrix} 2016-x & 2018+x & 2017-x \\ 2020-2 \cdot x & 2021+2 \cdot x & 2022-2 \cdot x \\ x & -x & x \end{bmatrix}$;
- l) $A = \begin{bmatrix} 1-x^2 & 0 & 0 & 0 \\ \ln x+1 & e^x-1 & 0 & 0 \\ e^x-1 & \operatorname{tg} x & \ln x & 0 \\ \sin x & \cos x & \operatorname{ctg} x & \sin(\pi \cdot x) \end{bmatrix}$;
- m) $A = \begin{bmatrix} 1 & -x & 1 & 1 \\ x & 1 & -1 & -1 \\ 0 & 1 & -1 & x \\ -1 & -1 & x & 0 \end{bmatrix}$;



DETERMINANTE REDA 2 I 3. REGULARNE MATRICE.

$$\text{n) } A = \begin{bmatrix} x & 1 & 0 & 1 \\ 1 & -x & 1 & 0 \\ 1 & 1 & x & 1 \\ 1 & 0 & 1 & x \end{bmatrix};$$

$$\text{o) } A = \begin{bmatrix} x-1 & 0 & -1 & 0 \\ 1-x & 0 & 1 & 1 \\ 1+x & -1 & -1 & 0 \\ x & 0 & 1 & -1 \end{bmatrix}.$$

10. Odredite istinitost sljedećih tvrdnji i objasnite svoje odgovore:

- a) Inverz regularne gornje trokutaste matrice je gornja trokutasta matrica.
- b) Inverz regularne donje trokutaste matrice je donja trokutasta matrica.
- c) Inverz regularne dijagonalne matrice je dijagonalna matrica.